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Underwater parametric transmission with a linear array

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ABSTRACT

The modeling of parametric transmission by means of the spatial Fourier formalism (also called the angular spectrum method) is recalled. The secondary field can be estimated at any distance of the antenna. Various geometries of the transmitter can be also taken into account, including elongated antennas that feature a wide aperture in one plane. The relevance of the theoretical model with such geometry is tested with a linear array used as a parametric transmitter in an underwater application. Numerical estimations are compared with experimental measurements. In addition, the expected slight variation of the parametric efficiency with the relative phasing of the primary signals is experimentally observed. © 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Parametric transmission takes advantage of the non-linear interaction of two primary beams generated by the antenna at the high frequencies v_1 and v_2 [1–3]. These interactions create secondary waves at all the linear combinations of the primary frequencies. The secondary beam of interest is at the difference frequency $v_{-} = v_2 - v_1$. The main benefit of the parametric transmission is thus that a narrow beam can be generated at a low frequency by means of a physical antenna whose size is not very large compared to the parametric wavelength λ_{-} . Another interest is the potential large frequency bandwidth that can be achieved with such transmitters. However, a significant drawback of parametric transmission is the poor efficiency of the non-linear conversion. In addition, a saturation phenomenon occurs when increasing the primary source levels, which reduces both the parametric gain and directivity. Hence, the design of an efficient parametric transmitter calls for a delicate balance of the parameters of the antenna with respect to the required characteristics of the secondary beam.

The parametric generation is a cumulative process. The wave at the difference frequency grows in the source volume made of the primary waves. The Westervelt model exemplifies this principle by considering perfectly collimated primary beams: the source volume is in fine interpreted as an end-fire array whose length is only limited by the linear absorption of the primary waves. The parametric directivity is thus proportional to the square root of the array length. In addition, the beam pattern is devoid of side-lobe.

Many other models are proposed for handling more realistic source distributions [4–8], i.e., by taking into account altogether

* Corresponding author. E-mail address: jacques.marchal@upmc.fr (J. Marchal). diffraction, attenuation and saturation phenomena. In most applications, the width of the beams is relatively narrow all around the main axis (e.g., sub-bottom profilers). The paraxial approximation is thus convenient to model such transmitters.

In the underwater domain, a classical application of parametric transmission is sub-bottom profiling: because of the large attenuation, only low frequency waves can penetrate the sediments; but narrow beams are also required to improve the profile resolution. Recent projects studied the feasibility to detect buried objects by means of high resolution imaging [9–15]. The idea is to combine the parametric transmission and the synthetic aperture sonar techniques. The parametric beam must be narrow in one direction to achieve the required resolution in elevation, and sufficiently large in the perpendicular direction to perform the synthetic aperture process. The parametric projector.

The spatial Fourier formalism is well suited to handle versatile geometries of transmitter, including elongated rectangles. A model based on this formalism has been previously developed to estimate the parametric field at any distance of the transmitter, taking also into account the saturation effect [16–19]. One used this model to study a linear array whose primary and parametric frequencies are respectively in the ranges 80–120 kHz and 10–40 kHz, and the angular aperture of the secondary beam at 20 kHz is around $2^{\circ} \times 10^{\circ}$. Extensive measurements have been performed in tank with a prototype system, and then compared with the simulations. The experiments gave also the opportunity to observe a phenomenon that was already foreseen by an analysis of the Burgers equation [20].

The modeling of parametric antenna by means of the Fourier formalism is summarized in Section 2. Experimental measurements and comparison with the model are presented in Section 3.





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2. Theory

A complete description of the parametric transmission model can be found in our previous paper [18].

2.1. Non-linear equation

The general non-linear second order equation reads in term of the acoustic potential ϕ :

$$\Box'\phi = \mathcal{S}(\phi). \tag{1}$$

 \Box' is the classical d'Alembertian operator in absorbing media:

$$\Box'(\cdot) = \nabla^2 \cdot -\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \frac{2}{c_0} \mathscr{L}\left(\frac{\partial \cdot}{\partial t}\right),\tag{2}$$

where c_0 is the sound speed in the fluid, and \mathscr{L} is a linear operator that describes different phenomena related to attenuation (e.g. thermoviscous attenuation or relaxations). From a practical point of view, this operator is often defined in the frequency domain by using the attenuation coefficient $\alpha(v)$:

$$\mathscr{L}(e^{j2\pi vt}) = \alpha(v)e^{j2\pi vt}.$$
(3)

In Eq. (1), *S* is a quadratic source term defined by:

$$S(\cdot) = \frac{1}{c_0^2} \frac{\partial}{\partial t} \left\{ (\nabla \cdot)^2 + (\beta - 1) \frac{1}{c_0^2} \left(\frac{\partial \cdot}{\partial t} \right)^2 \right\},\tag{4}$$

where β is the nonlinear coefficient of the fluid. In the quasi-linear theory, it is assumed that: (1) the primary waves $\phi_{1,2}$ obey the linear wave equation; (2) the secondary wave ϕ_{-} is created by the interaction of the only primary waves:

$$\Box'\phi_{1,2} = 0, \tag{5}$$

$$\Box'\phi_{-} = \mathbf{S}_{-}(\phi_{1},\phi_{2}),\tag{b}$$

It is thus assumed that the source level of the primary waves is low enough so that there is no saturation effect involved. Note that *S* in Eq. (1) addresses all combinations of frequencies, whereas the source function S_{-} in Eq. (6) is restricted to the interaction of the only primary waves that yields the difference frequency component.

For higher primary levels, the above assumptions are no longer valid. However, solving directly Eq. (1) is not practical. The primary waves do not obey the linear equation, but a simplification can be used to derive the parametric field ϕ_- : the evolution of the on-axis primary levels is tapered to take into account an extra-attenuation caused by the saturation effect, but the diffraction of the primary fields is still described with Eq. (5). It is also still assumed that the wave at the difference frequency is produced by the only interaction of the primary waves. Consequently, Eq. (6) is now replaced by

$$\Box'\phi_{-} = S_{-}(u_{1}\phi_{1}, u_{2}\phi_{2}), \tag{7}$$

where $u_1(z)$ and $u_2(z)$ are the normalized coefficients that quantify the extra-attenuation of the primary waves (at frequencies v_1 and v_2). They are obtained by solving the generalized Burgers equation [3,20,21] which is based on a pseudo 1-D model [22]. This equation reads in terms of normal velocity V:

$$\frac{\partial V}{\partial z} = -\mathscr{L}(V) - \frac{m}{z}V + \frac{\beta}{2c_0^2}\frac{\partial(V^2)}{\partial t}, \quad \text{with} \begin{cases} m(z < z_1) = 0\\ m(z_1 < z < z_2) = 1\\ m(z > z_2) = 1 \end{cases}$$
(8)

where the diffraction coefficient m is dictated by the Fraunhofer distances z_1 and z_2 of the rectangular antenna (computed at the

mean primary frequency); the initial condition is given by the normal velocities V_{01} and V_{02} at the surface of the antenna, at the primary frequencies v_1 and v_2 . Solving Eq. (8) yields the evolution with z of the velocities at all the linear combinations of these primary frequencies. The coefficients u_1 and u_2 are the ratios

$$u_i(z) = \frac{V_i(z)}{V_i(z, \beta = 0)},$$
(9)

where $V(z, \beta = 0)$ is the solution of Eq. (8) with $\beta = 0$ (pseudo 1-D linear propagation).

2.2. Spatial Fourier formalism

With the spatial Fourier formalism, the acoustic fields are described by means of the superposition of inhomogeneous plane waves [23,24]. In a reference plane Π_0 , the spectrum of a harmonic field $g(\mathbf{r}, t) = G(\mathbf{r}) \exp(-j2\pi v t)$ at frequency v is the spatial Fourier transform:

$$A_0(\mathbf{f}) = \iint_{\mathbf{q}\in\Pi_0} G(\mathbf{q}) \exp(-j2\pi \mathbf{f} \cdot \mathbf{q}) d\mathbf{q}.$$
 (10)

where **f** is the spatial frequency and **q** the vector describing the plane Π_0 (see Fig. 1). In the case of a linear propagation, the spectrum of *G* in the plane Π_z parallel to Π_0 is given by:

$$A_{z}(\mathbf{f}) = A_{0}(\mathbf{f}) \exp\left(jk_{z}z - \frac{k}{k_{z}}\alpha(v)z\right), \text{ with}$$

$$k = 2\pi v/c_{0} \text{ and } k_{z}(f) = \sqrt{k^{2} - 4\pi^{2}f^{2}}, \tag{11}$$

where $\alpha(v)$ is the coefficient of linear attenuation at frequency v. The field *G* can be derived in any plane Π_z by the inverse transform:

$$G(\mathbf{r} = \mathbf{m} + z\mathbf{e}_z)$$

= $\iint A_0(\mathbf{f}) \exp\left(jk_z z - \frac{k}{k_z}\alpha(v)z\right) \exp(j2\pi\mathbf{f}\cdot\mathbf{m})d\mathbf{f}.$ (12)

where \mathbf{e}_z is the unit vector along the *z*-axis, and **m** the current point in the plane Π_z (see Fig. 1).

In the farfield, the Fraunhofer approximation yields straightforwardly from the initial spectrum A_0 :

$$G(\mathbf{r}) = \frac{z}{j\lambda r^2} \exp(jkr - \alpha(v)r)A_0\left(\frac{\mathbf{m}}{\lambda r}\right).$$
(13)

2.3. Model of parametric transmission

Using the Fourier formalism, the nonlinear propagation is addressed as the superimposition of interactions between inhomogeneous plane waves. In addition, because of the nonlinearity, the



Fig. 1. Notation of coordinates with respect to the reference plane.

spectra in the planes Π_z are no longer simply related to the spectra in the plane Π_0 through Eq. (11). However, it is convenient to keep the formulation (12), provided the reference spectrum $A_0(z, \mathbf{f})$ depends now on the abscissa *z* of the observation plane. Hence, denoting $A_{0,v_-}^{(p)}(z, \mathbf{f}_-)$ the spectrum of the parametric field (in terms of pressure), the spatial Fourier transform of Eq. (7) leads to:

$$\begin{aligned} A_{0,\nu_{-}}^{(p)}(z,\mathbf{f}_{-}) &= -j\pi \frac{P_{01}^{*}P_{02}\beta\nu_{-}}{\rho_{0}c_{0}^{3}} \\ &\times \iint \mathscr{A}_{1}^{*}\left(\mathbf{f}_{1} = \frac{\nu_{1}}{\nu_{-}}\mathbf{f}_{-} + \mathbf{f}'\right)\mathscr{A}_{2}\left(\mathbf{f}_{2} = \frac{\nu_{2}}{\nu_{-}}\mathbf{f}_{-} + \mathbf{f}'\right) \\ &\times \int_{0}^{z} u_{1}^{*}(z)u_{2}(z)\exp(-(\alpha - j\Delta k)z)dzd\mathbf{f}', \end{aligned}$$

$$(14)$$

where ρ_0 is the fluid density, $P_{0i} = \rho_0 c_0 V_{0i}$ (*i* = 1, 2) are the equivalent pressures for each primary radiation at the antenna surface with normal velocity V_{0i} , $\mathscr{A}_{i=1,2}(\mathbf{f})$ is the Fourier transform of the aperture of each primary source in the reference plane Π_0 that contains the parametric antenna (* denotes the complex conjugate operator). The term $(\alpha - j\Delta k)$ is given by:

$$\Delta k = k_{2z} - k_{1z} - k_{-z}.$$
 (15)

$$\alpha = \frac{k_1}{k_{1z}(f_1)} \alpha(\nu_1) + \frac{k_2}{k_{2z}(f_2)} \alpha(\nu_2) - \frac{k_-}{k_{-z}(f_-)} \alpha(\nu_-) \quad (k_i = 2\pi\nu_i/c_0).$$
(16)

where $\alpha(v_i)$ are the attenuation coefficients associated to the primary and parametric waves. The value of the parametric pressure is then obtained by means of Eq. (12), or Eq. (13) in the farfield.

The interest of Eq. (14) is that the most constructive interactions occur from quasi collinear primary plane waves, for which Δk vanishes. It is a very useful property for the numerical implementation of this model. Such an implementation has been made for underwater applications [18,19,25,26], where the sound speed and attenuation of sea water are computed with the Lovett [27] and François–Garrison [28] empirical models, respectively.

3. Experimental measurements

3.1. Setup

The active face of the investigated linear array is 72 cm long and 6 cm large. It is divided into 48 elements. Each channel is driven by a linear power amplifier that delivers up to 50 W through a matching unit. The signals are provided by a digital synthesizer with versatile capabilities. The frequency band of the whole transmitter is in the range 80–120 kHz. The primary source level is around 237 dB re 1 μ Pa_{rms} @ 1 m. At 100 kHz, the half-power beam-width of the whole antenna is around 1° × 12°.

The measurements were conducted in a large tank that is part of the naval facilities in Brest. The tank is 80 m long, and 10 m wide, with a water column height of 8.4 m. The array is hung vertically from a deck across the tank, at mid-width and mid-depth. The suspension of the array includes a vertical axis of rotation. The 72 cm-long array is oriented along this axis, so that the 1°-aperture of the array is vertical (elevation angle) whereas the 12°-aperture is horizontal (azimuth angle). The receiving hydrophone (Reson TC4034) is deployed from another deck, at range varying between 10 m and 50 m from the transmitter.

Measurements in azimuth are performed with the manual rotation of the axis. The angular accuracy of the settings is sufficient because of the order of magnitude of the azimuthal apertures $(\sim 10^\circ)$. On the other hand, the diagrams in elevation are derived by steering electronically the beams (pure angular deviation, or focusing also at finite distance). This technique is justified because the total aperture in elevation of the antenna is small (primary ~1°, parametric ~2°), and the aperture in elevation of each element of the array is much larger (~50°). If physically rotated by a few degrees only, (1) the apparent length of the array would be almost a constant; (2) the shading effect caused by the elementary directivity would be negligible.

The calibration of the primary beams is performed at a low source level in order to avoid any saturation effect (-30 dB below max level). In Fig. 2, it can be observed that the experimental data collected in focusing the transmitter at the range of the hydrophone (20 m and 50 m) match perfectly the theoretical expected far-field pattern, both in the main lobe and the side lobes at around $\pm 2^{\circ}$. The diagram obtained at 50 m by only steering the beam (i.e., not focusing at finite distance) does not match as well, which shows that the farfield condition is not yet entirely fulfilled at this distance.

3.2. Results and Discussion

The parametric transmission was tested in a large number of configurations. We present only here a few significant results. Besides the calibration devoted to the particular application of this antenna, the interest is twofold with such an antenna whose aperture is very narrow in elevation and much larger in azimuth: (1) it exhibits the typical behavior of the parametric transmission with this geometry; (2) it gives the opportunity to compare experimental results with the model.

The parameters of the configurations are:

- the parametric frequency $v_{-} = v_2 v_1 (10-40 \text{ kHz}, \text{ step 5 kHz})$
- the parametric ratio $\mu = \frac{\nu_2 + \nu_1}{2\nu}$ (2.5–10.5, depending on the para-
- metric frequency)the distance of observation (10–50 m)

In order to ease the comparison with the model, the primary beams are only steered, i.e., not focused at a finite distance, to measure the parametric field. In addition, all the displayed parametric levels and diagrams are conventionally corrected for attenuation and spherical spreading to give values referenced at 1 m.

The parametric aperture in azimuth measured at 20 m (Fig. 3) does not vary much $(9-11^{\circ} \text{ at } -3 \text{ dB})$ with the frequency settings. As expected, it can be observed that the parametric efficiency increases when the parametric ratio μ decreases. The parametric aperture in elevation (Fig. 4) is much more dependent on the difference frequency $(1.6-2.4^{\circ})$. The mean difference in the apertures



Fig. 2. Primary diagram in elevation, at 100 kHz, 20 m and 50 m distance (vertical unit is scaled as sensitivities of the whole transmitting setup, i.e., power amplifier + transducer). Solid line: theory, marks: measurements.



Fig. 3. Parametric diagram in azimuth at 20 m (vertical unit is scaled as parametric source levels). Lines: model; marks: measurements (dashed lines correspond to empty marks, solid lines with filled marks).

derived from the experimental data and from the model is around 0.2° (difference is slightly larger at v_{-} = 10 kHz).

The cumulative process involved in the parametric conversion is clearly put in evidence with the evolution of the diagrams and levels with the distance (Figs. 5 and 6). The directivity in elevation is significantly better at 20 m from the transmitter than at 10 m: the variation in the longitudinal extent of the primary source is the main cause. Beyond 20 m distance, the beam pattern is stable. However, it can be observed that the parametric level (displayed as an equivalent source level at 1 m) continues to increase beyond 20 m. The primary waves are still enough energetic to feed the parametric radiation. Although the aperture in azimuth is quite large, the parametric efficiency remains around -40 dB. For example, the parametric source level at 20 kHz measured at 20 m is 195 dB re 1µPa_{rms} @ 1 m. This level is sufficient to penetrate a sediment at a few meters under the interface with the sea water.

At small parametric ratio, the relative phasing of the two frequency components of the primary signal may impact on the parametric efficiency. This is because the shape of the primary signal changes significantly with this phasing [20]. The experimental evidence of this phenomenon has never been reported in the literature. Fig. 7 displays the parametric on-axis level obtained at 20 m. The array is driven with the signal $s(t) \propto \sin(2\pi v_1 t) + \sin(2\pi v_2 t + \varphi)$, with the primary frequencies $v_1 = 80$ kHz and $v_2 = 120$ kHz so that $\mu = 2.5$. The variation of the parametric source level with the phasing φ increases with the primary source level. At the maximal transmitter power (SL = 237 dB), the amplitude of the fluctuation reaches 2 dB. This is not a very large change,



Fig. 4. Parametric diagram in elevation at 20 m (vertical unit is scaled as parametric source levels). Solid line: model, marks: measurements.



Fig. 5. Parametric diagram (20 kHz) in elevation at several distances (vertical unit is scaled as parametric source levels). Solid line: model, marks: measurements.



Fig. 6. On-axis parametric source level versus distance. Lines: model; marks: measurements (dashed lines correspond to empty marks, solid lines with filled marks).



Fig. 7. Parametric levels measured at 20 m (ν_{-} = 40 kHz with ν_{1} = 80 kHz, ν_{2} = 120 kHz) (vertical unit is scaled as parametric source levels) versus the relative phasing φ of the primary signals. Red curve: primary source level is maximal (237 dB re 1µPa_{rms} @ 1 m); Green curve: primary source level is -5 dB below max level. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

although not negligible. Note also that this phenomenon can also induce unexpected fading oscillations. For example, considering primary frequencies at $v_1 = 90$ kHz and $v_2 = 130$ kHz: the parametric frequency is still at 40 kHz; because nonlinear propagation creates waves at all linear combinations of the primary frequencies, sub-harmonics at frequencies multiple of 10 kHz are also generated in this case (e.g., with the combination 3×90 kHz- $2 \times$ 130 kHz). This may induce variations in the parametric efficiency with a cyclic recurrence at 10 kHz, i.e., every four parametric periods. The amplitude of the fading phenomenon should have the same order of magnitude as in the experimental example, i.e., about 2 dB. Hence, the primary signals used to drive a parametric transmitter with a small parametric ratio must be designed carefully to maximize the conversion efficiency and to avoid unwanted fluctuations.

4. Conclusion

The spatial Fourier formalism is a convenient tool to model the paraxial parametric transmission in the frame of nonlinear interactions between finite-amplitude waves. The secondary field can be obtained at finite distances, with versatile projector geometries. The numerical implementation of the theoretical model is manageable with reasonable efforts.

The validity of the model could not be a priori taken for granted with geometry that involves very dissymmetric apertures, e.g., linear arrays. However, the confrontation with experimental data shows good agreements.

The most questionable hypothesis in this finite-amplitude model is the separability of the distance and spatial frequencies that Eq. (7) involves. More specifically, the pending question is the limit of validity and the consistency of the 1-D model in estimating the extra-attenuation. The presented results shows that this limit is not reached with the primary source levels used in this application, which are already quite large.

An experimental evidence of the dependency of the parametric efficiency on the phasing of the primary signals is given. It shows that the phenomenon must not be neglected in applications calling for small parametric ratios, as needed to obtain a large parametric conversion efficiency.

At the present time, the model is being implemented to simulate the parametric radiation obtained with a Mills cross antenna (each arm transmits one of the primary frequency – the parametric beam can be steered by steering each primary beam). A future prospect is also to test this model in aerial applications for which the orders of magnitude are different than in underwater applications.

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