FRESNEL APPROXIMATION IN THE NON PARAXIAL CASE

AND IN ABSORBING MEDIA.

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INTRODUCTION

The Fourier theory of harmonic radiation is now a classical tool in Optics or in Acoustics {1}. It is in general limited to an isotropic non absorbing medium and most of its applications are obtained in the frame of the Fresnel (or Fraunhofer) paraxial approximation which remains valid for the great majority of optical or acoustical imaging or detection devices. It implies essentially a 2-D Fourier analysis in planes Π_Z normal to a mean propagation direction Oz. The purpose of this article is to recall {2,3,4,5} that such an analysis may be extended straight forward to absorbing media just by using adequate plane modes expressing a phase variation dictated by a classical real wave vector and a an attenuation dictated by an imaginary wave vector parallel to Oz in such a way that the equiamplitude planes remain parallel to the reference plane z = O ; the signature of such a "generalized plane wave" remains adequate for a Fourier analysis. It will be shown that the Fresnel approximation has also a straight forward extension in this case, so that all the classical results of the Fourier theory in the Fresnel frame may easily be corrected. Furthermore, the Fresnel approximation consists in using a limited development of the z component of the wave vector in terms of the spatial frequency $\vec{f}$ valid for small values of $\vec{f}$. It may be extended to non paraxial situations important in NDE or in submarine detection of objects buried in sediments occurring when a beam is focused through a plane interface at large incidence. The mean direction of the transmitted beam may encounter a large deflexion, but corresponds in fact to the same spatial frequency $\vec{f}_0$ in planes parallel to the interface than the mean incident direction in the first medium. It is then possible to extend the Fresnel approximation to limited developments in terms of $\vec{f}' = \vec{f} - \vec{f}_0$, an approximation which remains valid if the original beam has a reasonable angular aperture. The main aberrations encountered by the transmitted beam may be shown easily when assuming that the incident focus beam has a gaussian structure.

PLANE FOURIER ANALYSIS OF A LINEAR HARMONIC RADIATION IN AN HOMOGENEOUS MEDIUM

If the radiation field may be described by means of a scalar function $\phi(M,t) = U(M)e^{-j\omega t}$, a Fourier analysis of the complex amplitude U may be developed inside planes Π_z normal to a chosen direction Oz, with the family of functions U_z :

$$U_z(\vec{m}) = U(\vec{M}) \quad , \quad \vec{M} = \vec{m} + z\vec{z} \in \Pi_z$$

For the Fourier expression of $U_0(\vec{m})$ in Π_0 :

$$U_0(\vec{m}) = \int A_0(\vec{f})\, e^{2j\pi \vec{f}.\vec{m}} \, \vec{df} \tag{1}$$

the solution for the half space $z > 0$ in the absence of sources may be written :

$$U(\vec{M}) = \int A_0(\vec{f})\, u(\vec{M},\vec{f})\, \vec{df} \tag{2}$$

where $u(\vec{M},\vec{f})$ is the solution of the propagation equation which has the signature $e^{2j\pi\vec{f}\cdot\vec{m}}$ in Π_0.

For an homogeneous medium, the solution u is a "plane" wave $e^{j\vec{k}\cdot\vec{M}}$ where $\vec{k} = 2\pi\vec{f} + k_z\,\vec{z}$ must satisfy the dispersion equation associated to the propagation equation and the relation (2) gives :

$$U_z(\vec{m}) = \int A_0(\vec{f})\; e^{jk_z z}\; e^{2j\pi \vec{f}.\vec{m}}\, \vec{df}, \tag{3}$$

which means that the radiation in Π_z may be expressed from its expression in Π_0 through the operations in the real space and in the Fourier space :

$$A_z(\vec{f}) = A_0(\vec{f})\, H_{0z}(\vec{f}) \quad , \quad H_{0z}(\vec{f}) = e^{jk_z z}$$
$$U_z(\vec{m}) = U_0(\vec{m}) * h_{0z}(\vec{m}) \quad h_{0z}(\vec{m}) = TF^{-1}(H_{0z}) \tag{4}$$

In the classical Fourier theory of optics or acoustics, the propagating medium is isotropic and non absorbing and the propagation equation is the ordinary wave equation :

$$\Box\varphi = 0 \quad , \quad \Box \equiv \frac{1}{c^2}\frac{\partial_2}{\partial t^2} - \nabla^2 \tag{5}$$

The plane waves $e^{j\vec{k}\cdot\vec{M}}$ are ordinary plane waves :

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad , \quad k_z = \frac{2\pi}{\lambda}\left(1 - \lambda^2 f^2\right)^{1/2}. \tag{6}$$

The Fresnel paraxial approximation ($\lambda f < 1$) uses the classical expressions for the operators H_{oz} and h_{oz} :

$$\begin{aligned} H_{oz} &\simeq e^{2j\pi\frac{z}{\lambda}} \, e^{-j\pi\lambda z f^2} \\ h_{oz} &\simeq \frac{1}{j\lambda z} \, e^{2j\pi\frac{z}{\lambda}} . \, e^{j\pi\frac{m^2}{\lambda z}} \end{aligned} \tag{7}$$

THE CASE OF AN ATTENUATING MEDIUM

The preceding formalism is extended straightforward to the case of an absorbing medium, for example the acoustical radiation in a viscous fluid : the propagation equation may be written for the velocity potential ϕ :

$$\Box'\varphi = 0 \, , \quad \Box' \equiv \frac{1}{c^2}\frac{\partial_2}{\partial t^2} - \left(1 + \frac{\eta}{\rho_0 c^2}\frac{\partial}{\partial t}\right)\nabla^2 \tag{8}$$

The dispersion equation :

$$k'^2\left(c^2 - j\omega\eta/\rho_0\right) = \omega^2 \tag{9}$$

leads simply to attenuated plane wave modes $e^{j\vec{k}''\cdot\vec{r}}$, where $\vec{k}'' = k''\vec{n}$ and $k'' \cong k + j\alpha$:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad , \quad \alpha = \frac{\eta\omega^2}{2\rho_0 c^3}$$

α being the classical attenuation coefficient : an approximate solution valid for $\alpha\lambda \ll 1$.

These modes are not appropriate to a Fourier analysis and must be replaced by modes $u(\vec{f},\vec{M}) = e^{j\vec{k}'\cdot\vec{M}}$ of signature $e^{2j\pi\vec{f}\cdot\vec{m}}$ in Π_0 with a complex component k'_z of the wave vector $\vec{k}'$. These modes (Fig. 1) have equiphase planes normal to the real part of k' and equiamplitude planes normal to Oz, and are determined from the dispersion equation according to :

$$k_z'^2 = \frac{\omega^2}{c^2 - j\omega\eta/\rho_0} - 4\pi^2 f^2 \tag{9'}$$

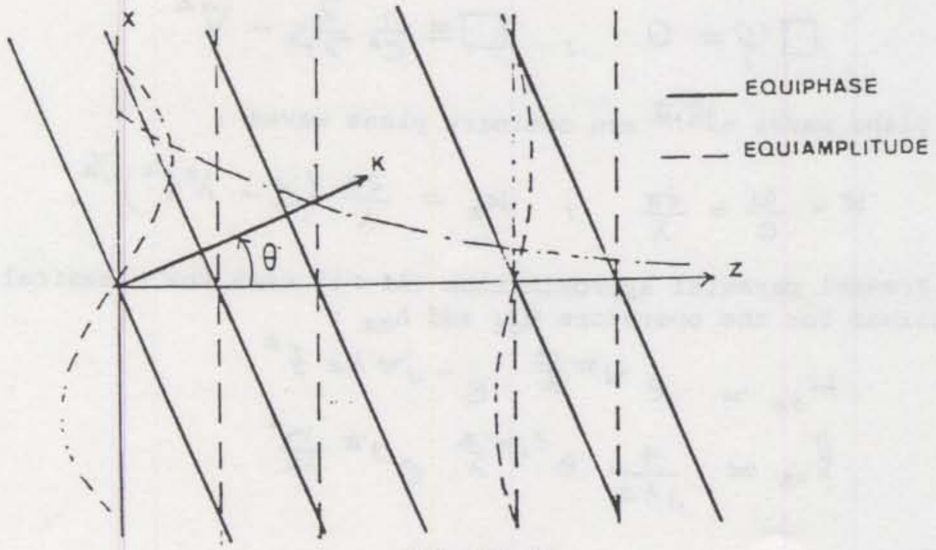

Figure 1

If $\alpha\lambda \ll 1$, the adequate solution may be written :

$$\vec{K}' = \frac{2\pi}{\lambda}\vec{n} + j\frac{\alpha}{\cos\theta}\vec{z} \quad , \quad \theta = (\vec{z}, \vec{n}) \tag{10}$$

and the corresponding Fresnel paraxial approximation ($\lambda f \ll 1$) to :

$$K'_z \simeq \frac{2\pi}{\lambda} - \pi\lambda f^2 + j\alpha\left(1 + \frac{\lambda^2 f^2}{2}\right) \tag{11}$$

The propagation operators from Π_0 to Π_z are then modified from (7) according to :

$$H'_{0z}(\vec{f}) = H_{0z} A_{0z} \; , \quad A_{0z} = e^{-\alpha z} e^{-\pi\left(\frac{\alpha z \lambda^2}{2\pi}\right) f^2} \; ,$$

$$h'_{0z}(\vec{m}) = h_{0z} * a_{0z} \; , \quad a_{0z} = \frac{2\pi}{\alpha z \lambda^2} e^{-\alpha z} e^{-\pi\left(\frac{2\pi}{\alpha z \lambda^2}\right) m^2} \; , \tag{12}$$

which means that the effect of the attenuation is in addition of a global attenuation $e^{-\alpha z}$ a smoothing by a gaussian function $e^{-\pi(m/\delta)^2}$. This smoothing effect is in general of negligible importance in practical imaging devices because δ remains very small in front of the focused spot size $\varepsilon = \lambda/\theta_0$, where θ_0 is the angular aperture of the focusing system. The equality $\varepsilon = \delta$ requires $\alpha z = 2\pi/\theta_0^2$ and a global attenuation of 55 dB for an aperture of $\theta_0 = 1$, which seems to be of consideration only for acoustical microscopy where both high angular apertures and high attenuations may be encountered {5}.

EXTENSION OF THE FRESNEL APPROXIMATION TO NON PARAXIAL BEAMS

It is well known that a focused beam oriented obliquely towards a plane interface separating two media of different velocities encounters strong aberrations. This situation interests the submarine detection of objects buried in sediments or, in N D E, the detection of defects not parallel to the surface. In this case, the Fourier analysis in planes Π_z parallel to the interface Π_0 remains very useful but the paraxial approximation is no longer valid. However, an equivalent approximation may be developed considering that the mean direction of the beam corresponds to plane wave modes associated to the same spatial frequency $\vec{f}_0$ in both media so that the spectrum $A_z(\vec{f})$ of a beam of reasonable aperture may be developed around $\vec{f}_0$, i.e. in terms of the spatial frequency $\vec{f}' = \vec{f} - \vec{f}_0$. The equivalent Fresnel approximation consists then in developing k_z in terms of $\vec{f}'$ and using the associated expressions for the propagation operators H_{oz} and h_{oz}, the dispersion equation associated to an isotropic medium gives through an elementary calculation, choosing $\vec{f}_0 = f_0 \vec{x}$:

$$K_z'^2 + 4\pi^2 f^2 = F(\omega) = K_{z_0}'^2 + 4\pi^2 f_0^2 ,$$

$$K_z' \simeq K_{z_0}' - 2\pi\left(\frac{2\pi f_0}{K_{z_0}}\right) f_x' - \pi\left(\frac{2\pi}{K_{z_0}'}\right)\left[\left(1+\frac{4\pi^2 f_0^2}{K_{z_0}'^2}\right) f_x'^2 + f_y'^2\right] , \qquad (13)$$

or, for the classical isotropic non absorbing medium :

$$K_z \simeq K_{z_0} - 2\pi \, tg\,\theta_0 \, f_x' - \pi \frac{\lambda}{\cos\theta_0}\left[\frac{f_x'^2}{\cos^2\theta_0} + f_y'^2\right] , \qquad (13')$$

where θ_0 is the incidence associated to the spatial frequency $\vec{f}_0$.

Obviously, the propagation operator $H_{oz} = e^{jk_z z}$, associated to the "Fresnel" expression of k_z, will deliver for a gaussian beam focused in Π_{z_0} $(z = z_0)$:

$$A_{z_0}(f_x', f_y') = C_0 \, e^{-\pi\left(b_{0x}'' f_x'^2 + b_{0y}'' f_y'^2\right)}$$

$$U_{z_0}(x,y) = \frac{C_0}{\sqrt{b_{0x}'' b_{0y}''}} \, e^{-\pi\left(\frac{x^2}{b_{0x}''} + \frac{y^2}{b_{0y}''}\right)} \, e^{2j\pi f_0 x} \qquad (14)$$

a solution in Π_z of the form :

$$A_z(f_x', f_y') = C \, e^{-2j\pi a f_x'} \, e^{-j\pi\left(b_x f_x'^2 + b_y f_y'^2\right)} , \qquad (15\text{-a})$$

$$U_z(x,y) = \frac{C}{\sqrt{b_x b_y}} \, e^{-j\pi\left[\frac{(x-a)^2}{b_x} + \frac{y^2}{b_y}\right]} \cdot e^{2j\pi f_0 x} , \tag{15-b}$$

where a, b_x, b_y are complex quantities, and $\sqrt{b_x b_y}$ the square root with a positive real part.

Noting $a, b_{x,y} = a', b'_{x,y} - ja'', b''_{x,y}$, one may check that the $b''_{x,y}$ must remain > 0, and that the focus F_0 corresponds to the nulling of the $b'_{x,y}$. If now this beam is focused through an interface located in Π_0 (z = 0) so that F_0 is a virtual focus, this formalism permits to attain easily the nature of the beam transmitted through the different material disposed in z > 0 and noted with the subscript 1 ;

$$A^1_z(\vec{f'}) = A_{z_0} \cdot H_{z,0} \cdot t \cdot H^1_{0z} , \tag{16}$$

where $t(\vec{f'})$ is the coefficient of transmission for generalized plane modes associated to the signature $\vec{f'}$ in both materials.

A problem arises from the transmission term t(f') which depends on the physical nature of the problem. For beams of low angular aperture, a reasonable approximation is to assume $t(\vec{f}) \cong t(\vec{f}_0)$ because $t(\vec{f})$ has slow variation in phase in front of $A_0(\vec{f}')$. We have also used matched developments of $t(\vec{f}')$ which have the mathematical form given in (15) for the A_z so that the gaussian structure of the beam remains preserved. This last operation affords negligible corrections on the numerical results obtained in the great majority of the problems that we have studied. In both cases, the solution $U^1_z(x,y)$ remains identical to the solution (15). An elementary calculation shows that the maximum radiation is located in Π_z at :

$$x = a' - a''(b'_x / b''_x) , \quad y = 0, \tag{17}$$

and the width of the beam at 6 dB in the directions x or y is :

$$\ell_x = .47\sqrt{b''_x + b'^2_x / b''_x} , \quad \ell_y = .47\sqrt{b''_y + b'^2_y / b''_y} \tag{18}$$

THE CASE OF NON ABSORBING MEDIA

This case interests many optical or acoustical situations. In acoustics, an interesting example is in N D E the detection of defects oriented obliquely or even normally to the surface of a solid. Such a detection may be done using an immersion technique and a focusing transducer oriented obliquely towards the surface with an adequate angle of incidence, so that the transmitted beam of transverse (or longitudinal) waves has sufficient incidence to give enough reflection from the target, specularly or by edge diffraction. In the solid medium, the shear wave (of the SV kind) may be described through a vector potential $\vec{\psi}^1 = \psi^1 \vec{y}$ which gives the velocity through the relation $V = \overrightarrow{\mathrm{Rot}}\, \vec{\psi}^1$ so that the velocity potential in water (first medium, $c = 1500$ m/s) is associated to the scalar ψ^1 in the second medium (1) (steel for example : $C_T^1 = 3200$ m/s) and the transfer coefficient $t(\vec{f})$ must be expressed in term of ψ^1/ϕ for a plane wave of signature $\vec{f}$. For longitudinal waves, the retained scalar is obviously the velocity potential ϕ^1 (in steel , $C_L^1 = 5900$ m/s).

In a first approach, it is reasonable to neglect the attenuation for both media, and referring to the expression (13') of k_z for non absorbing media, it may be checked that the solution (15) associated to a gaussian beam (14) is modified by the propagation operators H_{oz} and H_{oz}^1 according to (16) in keeping constant values for $b''_{x,y} = b''_{ox,y}$. Then, from (18), it appears that the minimum width of the beam, i.e. focusing, is obtained in the x (or y) direction for a value of z such that b'_x (or b'_y) = 0, and from (17), the focus point is located at the corresponding value of $x = a'(z)$. If we assume $t(\vec{f}) = t(\vec{f}_0) = t_0$, the solution A_z^1 given by (16) corresponds to the values of $a, b_{x,y}$ taking in account the expression (13') of k_z :

$$\left\{\begin{aligned} a(z) &= -z_0 \operatorname{tg}\theta_0 + z \operatorname{tg}\theta_0^1 \\ b_x(z) &= -\frac{\lambda z_0}{\cos^3\theta_0} + \frac{\lambda^1 z}{\cos^3\theta_0^1} - j\, b''_{ox} \\ b_y(z) &= -\frac{\lambda z_0}{\cos\theta_0} + \frac{\lambda^1 z}{\cos\theta_0^1} - j\, b''_{oy} \end{aligned}\right. \qquad (19)$$

There are two focal points F_x and F_y where the beam width is minimized in the x and y directions respectively. They are located on the mean refracted ray $x = a(z)$ at respective depths :

$$\left\{\begin{aligned} F_x &: \quad z_x = z_0 \frac{\lambda}{\lambda^1} \frac{\cos^3\theta_0^1}{\cos^3\theta_0}, \\ F_y &: \quad z_y = z_0 \frac{\lambda}{\lambda^1} \frac{\cos\theta_0^1}{\cos\theta_0}. \end{aligned}\right. \qquad (20)$$

Taking in account that $f_0 = \sin\theta_0/\lambda = \sin\theta_0^1/\lambda^1$, it is easy to check that F_y is on Oz like F_0 and that the focal distances IF_x and IF_y are in the ratio :

$$IF_x / IF_y = \left(\cos\theta_0^1 / \cos\theta_0\right)^2, \qquad (21)$$

ratio which is < 1 if $C^1 > C$. This aberration is well known and may be explained using a ray theory (6).

Another interesting remark is that at the focuses F_x and F_y, the beam exhibits in the x and y directions respectively exactly the same gaussian variation as the virtual beam at F_0.

NUMERICAL EXPERIMENTS

In the proposed numerical applications, we point out the characteristics of the refracted acoustic field for two kinds of interfaces with different patterns of the incident beam.

Results are exposed through equiamplitude lines in the incident plane (X,Z) and in the perpendicular cylindrical (quasi planar) surface which contains the line of maxima levels. This second network is projected upon a vertical plane (Y,Z), and also upon the interface (X,Y) if the refracted mean angle is large. The corresponding 6 dB X and Y resolutions and the maximal pressure level are drawn as functions of depth. All distances are given in wave lengths λ in the first medium.

The first example concerns the interface water/steel with such an angular incidence that only shear waves are generated in metal. We assume a constant transmitting coefficient in those computations and attenuation is neglected, except when displaying the X Y resolutions and maximal level in Figs. 2-B and 3-B. In this case, $C/C^1 = 0.459$, $\rho/\rho^1 = 0.127$. We retain for the incident angle the value $\theta_0 = 18.94°$ (the critical angle $\theta_C = 27.33°$). 6 dB-half width at virtual focused points are $\ell_x = \ell_y = 2\lambda$.

The figures 2-A and 2-B illustrate the case of an incident gaussian beam which is focused at the depth $Z_x = Z_y = 200\ \lambda$. The figures 3-A and 3-B show the effects fo the aberration correction obtained using a bifocused beam with $Z_x = 358\ \lambda$ and $Z_y = 200\ \lambda$ $(Z_x/Z_y = \cos^2\theta_0/\cos^2\theta_0^1)$. In the displays 2-B and 3-B of levels and resolutions, dashed lines (b) and (c) show the influence of the absorption in the second medium for respective attenuations of 0.5 and 1 dB/λ^1. These results show that, as mentioned earlier for a normal incidence, the resolution is very weakly affected by the attenuation.

The second example recovers characteristics of a model water/marine sediments (modelled as an absorbing liquid):

$$C/C^1 = 0.754 \quad ; \quad \rho/\rho^1 = 0.667 \quad ; \quad \alpha^1 = 0.5\ dB/\lambda^1$$

We simulate a square projector with sides of 11.8 λ located 20 λ above the interface. Computations concern two different incident angles : 48° and 40° (the value of the critical angle is $\theta_C = 49.84°$). In each case, the depth of the focus F_y is settled so that lateral resolution keeps a maximal value at a depth of 20 λ. We obtain :

$Z_y = 36.4\ \lambda$ with $\theta_0 = 48°$ $(\ell_y = 3.2\ \lambda)$

and $Z_y = 18.3\ \lambda$ with $\theta_0 = 40°$ $(\ell_y = 1.9\ \lambda)$

So, we retain three choices for the location of F_x :

- a unique virtual focus $Z_x = Z_y$ (Fig. 4) :
- Z_x chosen so that acoustic pressure is maximal at a depth of 20λ (Fig. 5); then, $Z_x = 310\ \lambda$ when $\theta_0 = 48°$ ($\ell_x = 18.5\ \lambda$) and $Z_x = 63\ \lambda$ when $\theta_0 = 40°$ ($\ell_x = 4.1\ \lambda$)
- in the incident plane (X,Z), the beam is collimated (Fig. 6).

We can notice, from these results, that, in case of absorbing media, the nearer of the interface the virtual point F_x is, the more curved the maxima pressure line is, and the deeper its direction is oriented. On the other hand, maximizing the level at the depth of 20 λ leads to a quasi-uniform field in the X direction.

CONCLUSION

We have extended the classical Fresnel approximation to beams propagating in absorbing media and according to an oblique axis, a theory which permits to attain aberrations encountered when focusing through a plane interface. N D E and submarine acoustical examples have been treated for gaussian beams which give the behaviour of any focused beam at least for the main lobe.

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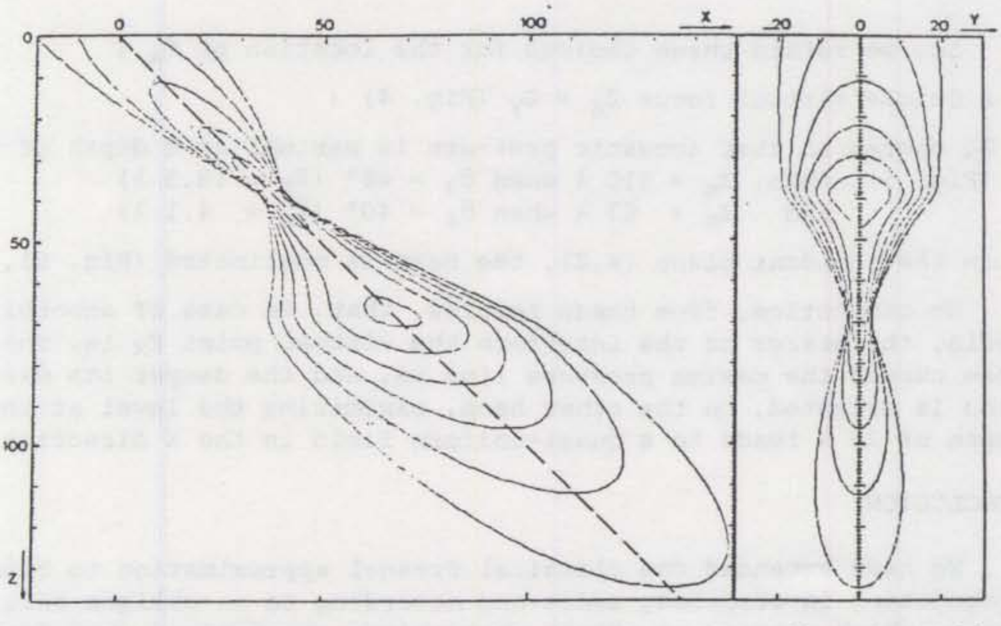


Figure 2-A - Equiamplitude lines : - 0.1, - 3, - 6, - 9, - 12 and - 15 dB ref. maxi pressure in the refracted field.

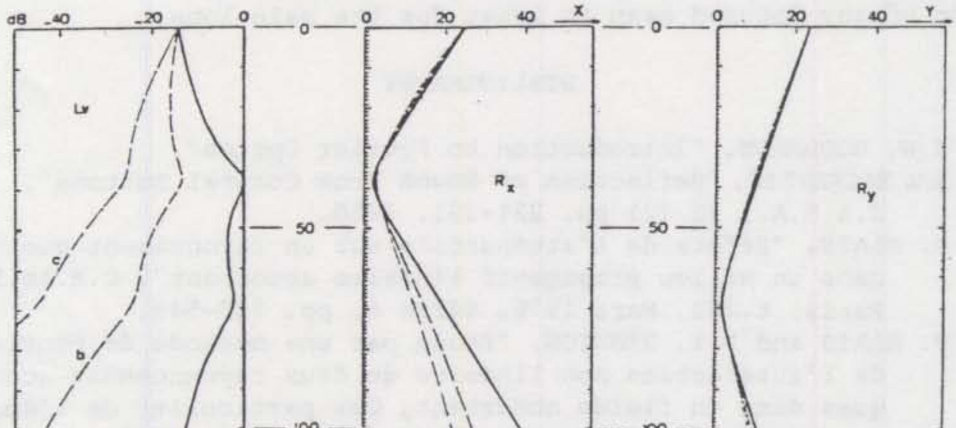


Figure 2-B - Maximal level (Lv) and X-Y resolutions (R_x and R_y) versus depth :
(a) $\alpha^1 = 0$ dB/λ^1 ; (b) $\alpha^1 = 0,5$ dB/λ^1 ; (c) $\alpha^1 = 1$ dB/λ^1

INTERFACE WATER/STEEL

$c/c^1 = 0.459$; $\rho/\rho^1 = 0.127$; $\theta_0 = 18.94°$
Unique virtual focus : $Z_x = Z_y = 200\ \lambda$; $\ell_x = \ell_y = 2\lambda$

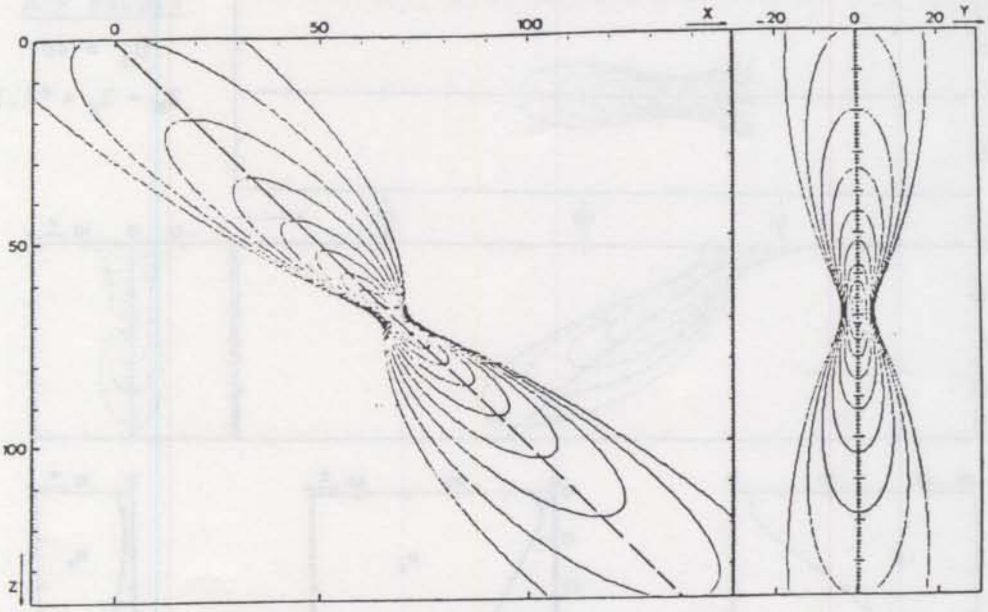


Figure 3-A - Equiamplitude lines : - 0.1, - 3, - 6, - 9, - 12 and - 15 dB ref. maxi pressure in the refracted field.

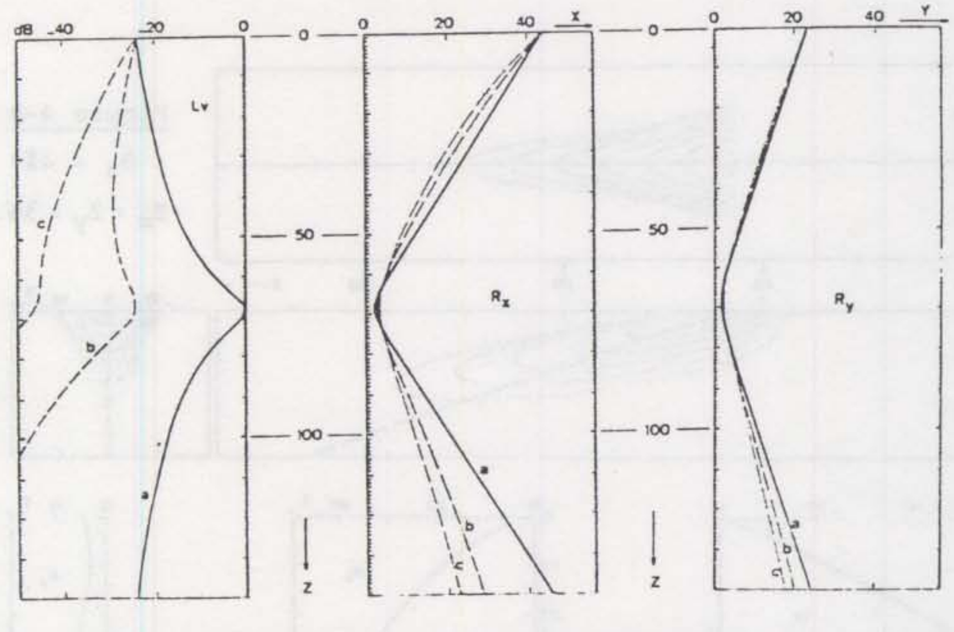


Figure 3-B - Maximal level (Lv) and X-Y resolutions (R_x and R_y) versus depth :
(a) $\alpha^1 = 0\ dB/\lambda^1$; (b) $\alpha^1 = 0{,}5\ dB/\lambda^1$; (c) $\alpha^1 = 1\ dB/\lambda^1$

INTERFACE WATER/STEEL

$c/c^1 = 0.459$; $\rho/\rho^1 = 0.127$; $\theta_0 = 18.94°$
Virtual focuses : $Z_x = 358\ \lambda$; $Z_y = 200\ \lambda$; $\ell_x = \ell_y = 2\ \lambda$

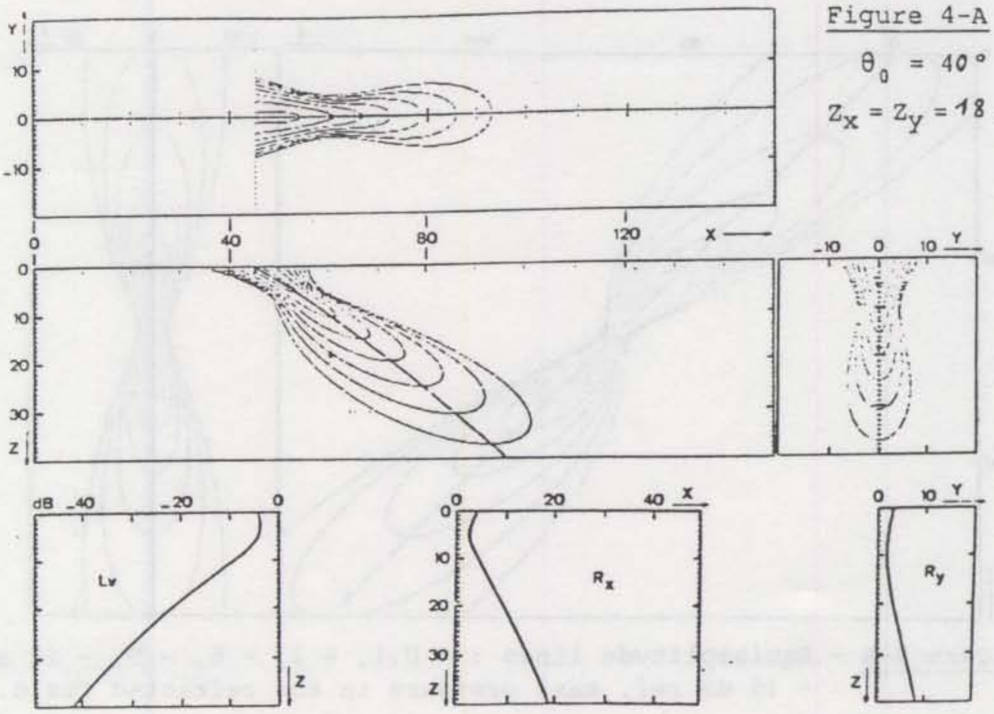


Figure 4-A

$\theta_0 = 40°$

$Z_x = Z_y = 18.3\,\lambda$

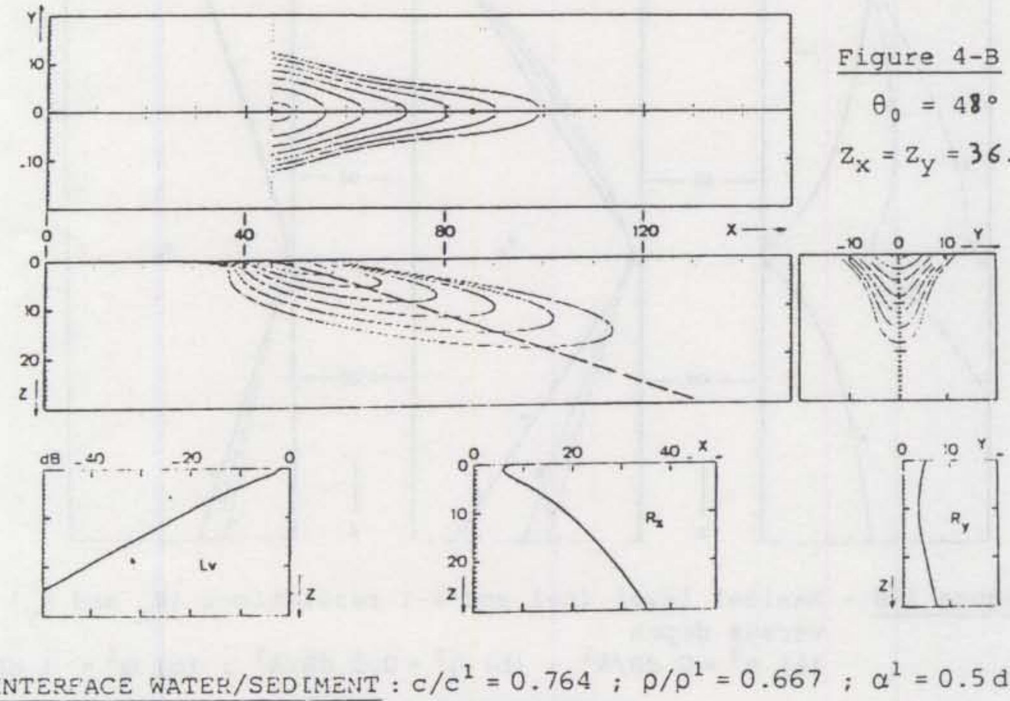


Figure 4-B

$\theta_0 = 48°$

$Z_x = Z_y = 36.4\,\lambda$

INTERFACE WATER/SEDIMENT : $c/c^1 = 0.764$; $\rho/\rho^1 = 0.667$; $\alpha^1 = 0.5\,\mathrm{dB}/\lambda^1$

- Transmitter : $11.8\,\lambda \times 11.8\,\lambda$ located $20\,\lambda$ above interface.
- Equiamplitude lines : - 1, - 6, - 12, - 18, - 24, - 30 and - 36 dB ref. maxi pressure in the refracted field.
- Maximal pressure level (Lv) and X-Y resolutions (R_x and R_y) versus depth.

Figure 5-A

$\theta_0 = 40°$

$Z_x = 63\,\lambda$
$Z_y = 18.3\,\lambda$

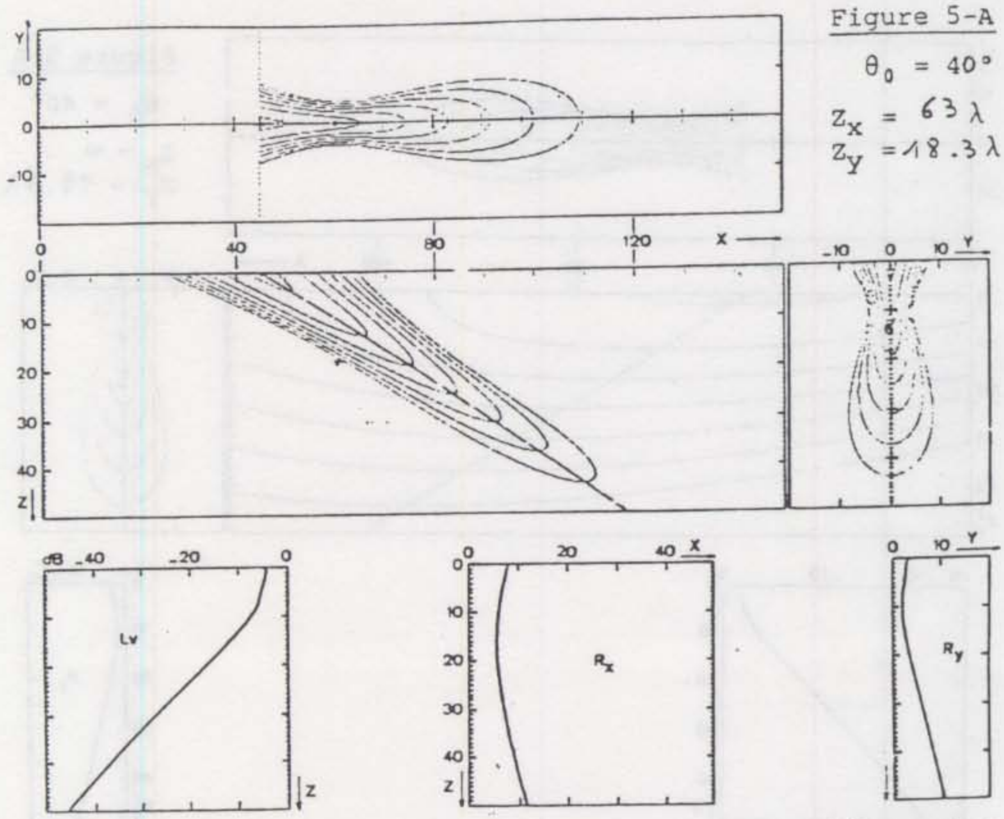
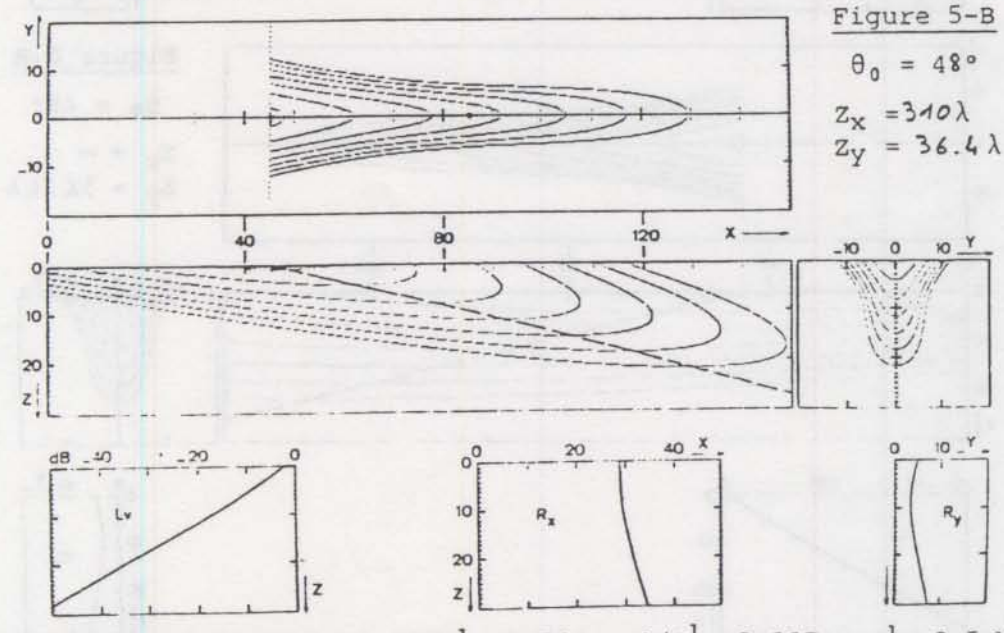


Figure 5-B

$\theta_0 = 48°$

$Z_x = 310\,\lambda$
$Z_y = 36.4\,\lambda$



<u>INTERFACE WATER/SEDIMENT</u> : $c/c^1 = 0.764$; $\rho/\rho^1 = 0.667$; $\alpha^1 = 0.5\ \mathrm{dB}/\lambda^1$

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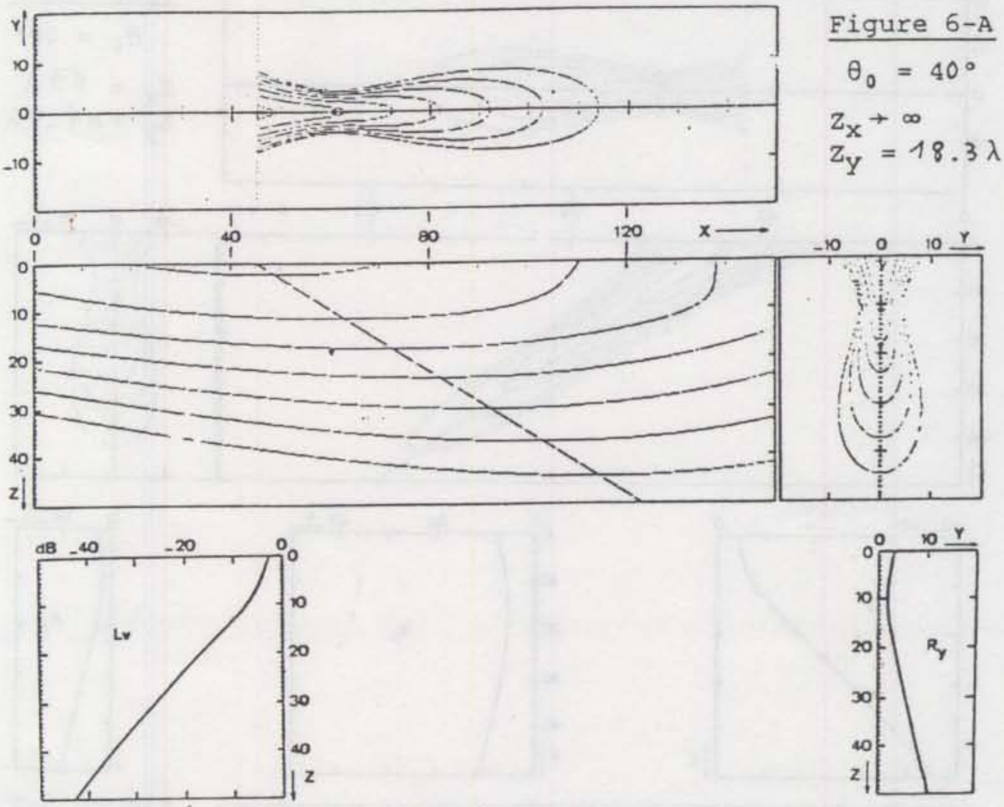

Figure 6-A

$\theta_0 = 40°$

$Z_x \to \infty$

$Z_y = 18.3\,\lambda$

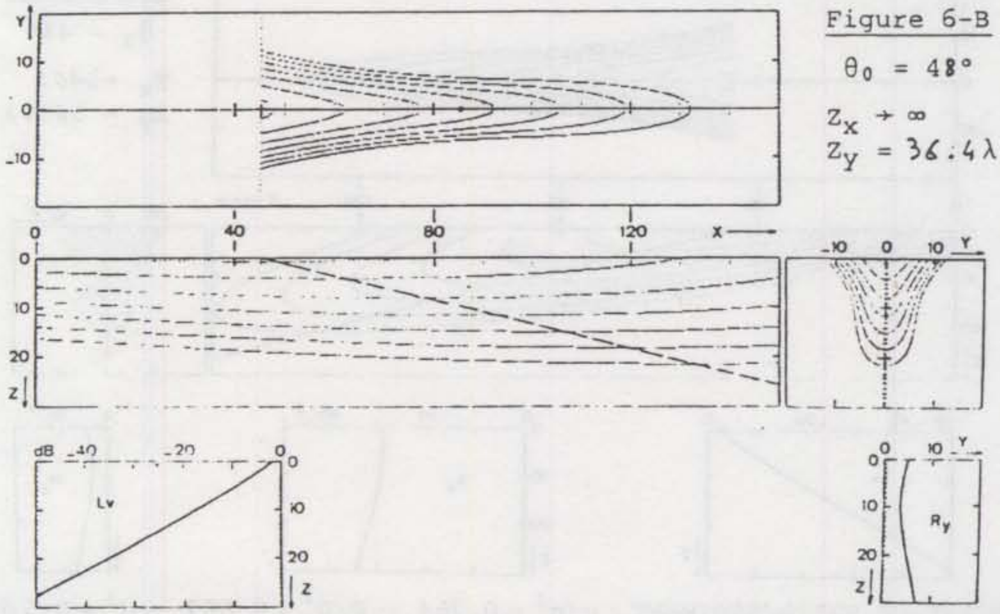

Figure 6-B

$\theta_0 = 48°$

$Z_x \to \infty$

$Z_y = 36.4\,\lambda$

INTERFACE WATER/SEDIMENT : $c/c^1 = 0.764$; $\rho/\rho^1 = 0.667$; $\alpha^1 = 0.5\,\mathrm{dB}/\lambda^1$

- Transmitter : 11.8 λ × 11.8 λ located 20 λ above interface.
- Equiamplitude lines : - 1, - 6, - 12, - 18, - 24, - 30 and - 36 dB ref. maxi pressure in the refracted field.
- Maximal pressure level (Lv) and Y resolution (R_y) versus depth.