A hybrid method for calibrating acoustic arrays

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Abstract — A method to calibrate the elements of large arrays devoted to underwater applications is presented. The goal is to measure the sensitivity and directivity of the elements over their full bandwidth. The main constraint comes from the bounded geometry of the experimental setups that limits the duration of the time windows available for analyzing the received signals. Using short wideband pulse is detrimental to obtaining high signal-to-noise ratios. A classical method for handling this problem is time-delay spectrometry (TDS), which is based on the transmission of a linear frequency-modulated signal combined with a sliding frequency filter. An alternative, hybrid method based on the transmission of a sequence of time-frequency-limited signals is proposed. This hybrid method is shown to provide the same spectral density as TDS in the frequency scanning, but the filtering process is quite different. The transmitted signals are designed to take advantage of the coherent sums of the received signals to track the time of flight of the direct paths between the source and the elements. In addition, a fitting process based on the calibration geometry of data acquisition enables the boundaries of the interference-free time windows to be precisely delineated. An example of the application is described.

Index Terms — calibration of array elements, time delay spectrometry, time-of-flight estimation.

I. INTRODUCTION

The correct tuning of underwater acoustical surveying tools (e.g., mine warfare sonar systems and multibeam echosounders) relies on the precise determination of the directivity and sensitivity of the transducer elements. Hence, many efforts have been made to improve the accuracy of the sonar calibrations achieved in a tank (e.g., [1] and [2]). This paper addresses the characterization of a receiving array using an external source. Adverse but common measurement environments are limited tank boundaries, large-sized arrays to be characterized, and limited telemetry. In such a context, a method is presented to derive the directivity and sensitivity of array elements over a wide angular sector and large bandwidth properly.

The geometry of the experimental setup raises several practical difficulties. The size of the arrays used in underwater applications can be relatively large. Because it is advantageous to make the calibration in the far-field area, there must be a sufficient distance between the source and the array. It is also convenient to limit the parallax biases: the post-processing stage must compensate for these differences in the angles of view from the different elements of the array to the source. On the other hand, the source levels used for calibration purpose must be large enough to ensure accurate measurements. Hence, directional sources are more efficient than omnidirectional hydrophones. Although the source far-field condition is usually easy to meet, one expects also that the acoustic field would be as uniform as possible over the entire extent of the receiving area. Hence, the use of directional sources tightens the distance requirement.

Given the distance between the source and the receiving array, there is a limited delay between the arrival of the signal through the direct path and the next arrival of the signal coming from the first reflection from the walls of the tank or the free surface. This delay decreases as the distance between the source and the receiver increases, thereby reducing the size of the time window available to process the direct signal without interference. Hence, measurement range and length of the time window are competing parameters.

The accuracy in the estimation of the times of flight is critical whenever the calibration
measurements are constrained by the size of the time window. If the geometric parameters of the experimental setup are perfectly known, the receiving time windows can be straightforwardly derived. However, accurate distance measurements and positioning of objects can be challenging in a large tank, depending on the sophistication of the facility. In addition, the time of flight can also vary with the orientation of the array much more than the signal durations when measuring the directivity of the elements. Element positioning and related issues are addressed in [3][4][5].

Several calibration methods have been in used for many years. A straightforward method consists of transmitting short wideband pulses [6]. The small duration of the pulse makes it easier to discriminate between the direct and reflected waves. The received signals are processed with the Fourier transform technique to determine the frequency transfer functions. The drawback of using a short wideband pulse is that the signal-to-noise ratio in each frequency band may be low because of the limited amplitude of the transmitted signal. There are several alternative solutions to this problem.

A classical solution is the gated method [7]; also called the tone burst method [8]. It consists of making a series of measurements with narrow-band quasiharmonic signals, which browse the whole frequency range with successive pings. The response at each frequency is measured on the steady part of the received signals. It is, indeed, mandatory that the stationary part can be clearly identified before any overlap with the reflected echo. Consequently, the duration of the available time window must be much larger than the duration of the initial transient part in the recorded signals.

Another technique is time-delay spectrometry (TDS) [9][10][11][12]. The source emits a linear frequency-modulated signal. Assuming that the time of flight is well determined, the received signal is band-pass filtered through a sliding time window. The central frequency of the filter shifts in time to follow the direct incoming wave and excludes multiple echoes with frequencies that do not match. Given the frequency sweep rate of the chirp, the parameters of the filter (bandwidth and duration) are proportional to the time delay between the direct and the reflected paths. The main concern with this method is that the frequency band of the received signal that is to be rejected is always adjacent to the band-pass domain, which raises a critical filtering issue.

We propose an alternative technique to TDS for window sizes too short to apply the gated method. As with the gated method, measurements are performed with a set of signals whose central frequencies sequentially cover the entire bandwidth of interest. However, the bandwidth of each individual signal is increased, but just enough that the duration fits with the available time window. Hence, the records can be clipped to process the entire direct signals received and derive the transfer functions properly. Compared with the single-pulse method, the frequency spectrum of each individual signal is narrower than the full spectrum of the transducer analysis, thereby improving the signal-to-noise ratio within each elementary band. It is therefore a hybrid method between the gated and impulse methods. The advantage compared with the TDS method is that this hybrid method overcomes the problems associated with the rejection of close interfering echoes. With both techniques, the analyzing time windows must be accurately delineated. To determine the time of arrival of the short transmitted signals, the proposed technique relies on the coherent phasing of the transmitted signals sequence. The times of flight are derived for all of the array elements, regardless of the array orientation. These sets of delays can be exploited to determine less well-known geometric parameters of the experimental setup. In turn, it allows refining of the position of the analyzing windows to compute the transfer functions with interference-free signals. The resulting calibration also benefits from this process because of a better knowledge of the geometric parameters, e.g., the accurate orientation of the array.

The hybrid method is presented in Section II with the estimation of the time of flight, the refinement of the geometric parameters, the final signal processing, and the comparison with TDS. An example of the implementation is described in Section III. The geometry of the experimental setup is first addressed; then parallax and delay between the direct path and the surface reflection are analyzed before describing the transmitted signals. The location of the reference hydrophone is discussed. The processing scheme is addressed next. It includes the search for the wavefronts and the retrieval of accurate geometric parameters before displaying the results. Finally, a comparison between the hybrid and classical methods is summarized in the conclusion.
II. Method

A. Time-of-flight estimation

An infinite sum of equal-amplitude sinusoids covering all frequencies, with proper phasing, is equivalent to a Dirac distribution function. Less drastically, the sum of a finite number of sinusoids spanning a finite bandwidth, when properly phased, results in a peaked function. One takes advantage of this property to track the direct signals received by the elements to be calibrated. A set of gated harmonic signals is designed to cover the whole frequency bandwidth of the analysis. These signals are all centered in phase. They are transmitted sequentially by the source. As with the gated method, a sufficient time delay must be planned between successive pings to avoid the presence of parasitic reverberation in the direct received signals. To find the time of flight between the source and one element (i.e., at a fixed location), the coherent sum of all the signals received by this element at the different frequencies is computed during post-processing. Provided the digital recordings are properly synchronized by the ping transmissions, the envelope of this sum exhibits a narrow peak that is easy to detect. This method is comparable to the technique used in [13] to detect direct-path arrivals in shallow water (i.e., sum of correlograms). There are many other methods for time-of-flight estimation (e.g., [14]), but this technique is particularly simple to implement, and provides sufficient accuracy for the required discrimination task.

To be more specific, consider a transducer to be calibrated over its bandwidth $\Delta f$ around the mean frequency $f_0$. It is mandatory that the duration $\Delta t_w$ of the time window that is available between the direct path and the first reflection is larger than the characteristic duration $(\Delta f)^{-1}$ of the impulse response of this transducer; otherwise, any transmitted pulse within the bandwidth of interest would produce the overlapping of the responses to the waves coming via the direct and indirect paths. Actually, $(\Delta t_w)^{-1}$ is the best frequency resolution that can be obtained in these conditions. In the following, it is assumed that the product $\Delta f/\Delta t_w$ is significantly larger than unity.

The set of $n$ signals to be transmitted is defined as

$$e_k(t) = a(t)\sin(2\pi f_k t), \quad \text{with} \quad f_k = f_0 + \left(\frac{k-1}{n-1} - \frac{1}{2}\right)\Delta f \quad k = 1, \ldots, n. \quad (1)$$

$a(t)$ denotes a modulating envelope (i.e., the upper limit of its spectral domain is much smaller than $f_k$), centered around $t = 0$, whose duration is $\Delta t_e$. The inverse $(\Delta t_e)^{-1}$ dictates the lower bound of the frequency bandwidth of these signals. Consistent with the relatively narrow-bandwidth approach, $\Delta t_e$ is thus chosen as large as possible compared to $(\Delta f)^{-1}$, although it remains smaller than the time window $\Delta t_w$ that warrants no interference with surface- or tank-reflected echoes:

$$(\Delta f)^{-1} \ll \Delta t_e < \Delta t_w. \quad (2)$$

The entire frequency domain $\Delta f$ is oversampled by choosing a large number $n$ of signals; the step $\delta f = \Delta f/(n-1)$ must be smaller than the bandwidth $(\Delta t_e)^{-1}$ of the transmitted signals to complete a gapless characterization of the transducer within the achievable frequency resolution:

$$\delta f \Delta t_e \ll 1. \quad (3)$$

Because $n \approx \Delta f/\delta f$, conditions (2) and (3) can be combined into

$$1 \ll \Delta f/\delta f \ll n. \quad (4)$$
Note that all the signals \( \{e_k\} \) are set with a null value at \( t = 0 \). The coherent sum of these signals reads

\[
\bar{e}(t) = n^{-1} \sum_{k} a(t) \frac{\sin(n \pi \delta f t)}{n \sin(\pi \delta f t)} \sin(2\pi f_0 t).
\] (5)

Taking into account the limited duration of \( a(t) \), condition (3) leads to the approximation:

\[
\bar{e}(t) \approx a(t) \sin(\pi \Delta f t) \sin(2\pi f_0 t).
\] (6)

Hence, the sum (5) consists of a carrier at the mean frequency \( f_0 \), whose modulating envelope is the cardinal sine function \( \text{sinc}(\pi \Delta f t) \), i.e., the peak width is commensurate to \( (\Delta f)^{-1} \), hence it is much smaller than \( \Delta t_w \).

B. Geometric parameters

The objective is to calibrate transducer elements of an array, i.e., to determine the angular sensitivities of the elements over the bandwidth of interest. Hence, measurements are performed at various orientations of the array. The relative positions of the elements inside the array are normally perfectly known. Note that the array is not necessarily linear. However, several other geometric parameters of the experimental setup (e.g., source-array distance and angular offset) are possibly not known with the required accuracy. Consider, for example, a source and a receiver located at the same depth \( h \) below the surface, at the horizontal distance \( \Delta d \). If the free surface is the closest reflecting plane, the difference \( \Delta d \) between the direct path and the reflected path defines the available time window:

\[
\Delta t_w = \Delta d/c \quad \text{with} \quad \Delta d \approx 2h^2/d,
\] (7)

where \( c \) is the speed of sound in water. For \( h = 1.5 \text{ m} \), and \( d = 30 \text{ m} \), \( \Delta d = 15 \text{ cm} \), i.e., 10 wavelengths at 100 kHz. In that case, one must know the length of the direct path with wavelength-order accuracy. Note that changes in the orientation of the array can induce variations in the distance between the source and the elements that can be larger than the window size \( \Delta t_w \).

One advantage of the proposed technique is that accurate times of flight are derived from the acoustic signals themselves. For each angular setting, all the signals received by the elements during the whole sequence of transmission are recorded. After summation on the different frequencies, the times of flight (versus elements orientation) are mapped in 2-D. This process is robust enough to safely identify the arrivals from the direct paths. However, fluctuations that the geometry of the setup cannot account for are likely to occur. The main reason is that the phase of the transfer functions between the source and the receiving elements varies with the frequencies and the relative orientations. Hence, the location of the detected peaks may vary with respect to the ideal mapping associated with the distances between the elements and the source. A worse situation occurs for the orientations of the array involving poor or even missing frequency contributions, e.g., between the main lobe and the first side lobe of elementary directivity diagrams. In these instances, the peak detection is still efficient because the missing values concern only part of the scanned bandwidth (given the orientation). However, the positions of the peak so determined this way are not accurate. The mapping errors can induce an incorrect adjustment of the windows that are eventually used to frame the records before processing. The quality of this selection is critical because, as exposed above, the size of the windows must be as large as possible to encompass the entire direct signal, although not encroaching upon the next wavefronts corresponding to reflected echoes.

The 2-D mapping used to determine the processing windows must be consistent with the geometry of the acquisition. The best way to accommodate the acoustical measurements with the experimental conditions is to fit the first-guessed 2D mapping with a geometrical model. This allows the less well-
known geometric parameters to be deduced. The classical way to proceed is to use the least-squares model for fitting. Depending on the context, it may be also possible to individually adjust each less well-known parameter, as shown in the example in Section III-B2. In any case, the resulting final model gives the positions of the windows actually used to crop the records. In addition, the refined geometry gives the proper angles and the distances needed to calibrate the elements.

C. Signal processing

Consider a single element with a given orientation of the array. It is expected that all the direct signals received at the different frequencies fit in the selected time window without any interference. The simplest way to derive the frequency response of the element is to extract the Fourier component of each record $s_k$ at the central frequency $f_k$ of the corresponding transmitted signal $e_k$. Practically, the following integral is computed for each received signal $s_k$:

$$S(\omega_k) = \int w(t) \tilde{s}(t) \exp(-j\omega_k t) dt \quad (\omega_k = 2\pi f_k), \quad (8)$$

where $\tilde{s} = s + j\hat{s}$ denotes the analytical signal derived with the Hilbert transform $\hat{s}$ of $s$, and $w(t)$ is the boxcar function corresponding to the time window whose width is $\Delta t_w$. $A(\omega)$ is the Fourier transform of the envelope $a(t)$ of the transmitted signal $e(t)$ [see (1)], and $F(\omega)$ the transfer function between the input of the transmitter and the output of the receiving element. Because the bandwidth of $a(t)$ is much smaller than $f_k$, (8) leads to

$$|S(\omega)| = |A(0)F(\omega)|. \quad (9)$$

Because $a(t)$ is user defined, $A(0)$ is perfectly known; however, $F$ splits into the product of the transfer function $T_F$ of the transmitter, the propagation operator $P_F$ that accounts for the geometrical spreading and the attenuation losses, and $R_F$, the transfer function of the element whose sensitivity (i.e., the magnitude $|R_F|$) is to be determined:

$$F(\omega) = T_F(\omega)P_F(\omega)R_F(\omega). \quad (10)$$

The time of flight $\Delta t$ between the source and the element is known with a sufficient accuracy to compute the magnitude of $P_F$:

$$|P_F(\omega)| = \frac{\exp\left(-\alpha(\omega)c\Delta t\right)}{c\Delta t}, \quad (11)$$

where the attenuation coefficient $\alpha$ and the sound speed $c$ depend on easily accessible parameters, e.g., temperature and salinity.

There are two options to deal with the transmitter sensitivity. When the magnitude $|T_F|$ of its transfer function is known, one can proceed directly with:

$$|R_F(\omega)| = \frac{|S(\omega)|}{|A(0)T_F(\omega)P_F(\omega)|}. \quad (12)$$

Whatever the source, an alternative, convenient way is to install a control hydrophone near the array to be calibrated. The signals $s_{\text{ref}}(t)$ recorded with this hydrophone are processed the same way as the elements of the array, hence giving the integrals $S_{\text{ref}}(\omega)$, whose magnitudes are equal to

$$|S_{\text{ref}}(\omega)| = |A(0)T_F(\omega)P_F(\omega)R_F(\omega)|. \quad (13)$$
Because the receiving sensitivity $|F_R^\text{ref}(\omega)|$ of the hydrophone is known, the only responses and propagation operators of the elements and of the control hydrophone are needed to derive the sensitivities $|F_R|$ of the elements:

$$
|F_R(\omega)| = |F_R^\text{ref}(\omega)| \frac{F_P^\text{ref}(\omega)}{F_P(\omega)} \frac{S(\omega)}{S^\text{ref}(\omega)}.
$$

(14)

Such a differential protocol makes the derivation independent of $|A(0)F_T(\omega)|$, i.e., not prone to biases because of an insufficiently accurate knowledge of the whole transmitting chain.

\section*{D. Comparison with time delay spectrometry (TDS)}

Whatever method is being used, the frequency resolution $\Delta f_e$ that can be achieved with a time window of analysis whose duration is $\Delta t_e$ ($<\Delta t_u$) is limited by the universal inequality

$$
\Delta f_e \Delta t_e \geq 1.
$$

(15)

On the other hand, the best strategy to improve the signal-to-noise ratio in the measurement is to extend by as much as possible the total duration of the signals transmitted to sample the entire frequency bandwidth of interest, $\Delta f$. In this respect, the worst solution is to use a single impulse signal whose bandwidth is $\Delta f$ (and characterizing the transducer by processing the received signal using of a Fourier transform). In the opposite trend, the gated method can be impractical if the time window $\Delta t_u$ is too small compared with the minimum delay $(\Delta f_e)^{-1}$ required to record the stationary part of narrowband tone bursts. The classical method to handle this problem is TDS. The total bandwidth is scanned with a linear frequency-modulated chirp characterized by the frequency sweep $\Delta f$ and the total duration $T$. The filtering process divides the signal into pieces of length $\Delta t_e$ ($<\Delta t_u$) during which the frequency sweep is $\Delta f_e$ :

$$
\Delta f_e = \frac{\Delta t_e}{T} \Delta f.
$$

(16)

Because of (15), the total duration of the chirp is thus limited by

$$
T \leq \Delta f (\Delta t_e)^2 \left( < \Delta f (\Delta t_u)^2 \right).
$$

(17)

Let us consider the total duration of the successively transmitted signals with the hybrid method. It is important to note that the comparison must consider a sequence of signals whose frequency bands do not overlap, with a bandwidth that is commensurate to the frequency resolution $\Delta f_e$. Consequently, the number of signals ($m$) to take into account is smaller than $n = \Delta f / \delta f$ :

$$
m = \frac{\Delta f}{\Delta f_e} \leq \Delta f / \delta f.
$$

(18)

The total duration of these concatenated signals $m\Delta t_e$ holds the same upper limit as in (17). From the energetic point of view, the hybrid method is thus equivalent to TDS. Compared to the shortest impulse response achievable with the bandwidth $\Delta f$, the order of magnitude of the time expansion provided by both methods is the classic time-frequency product

$$
T \Delta f \leq (\Delta f / \Delta t_e)^2.
$$

(19)

Note also that the characteristic duration of the sum of signals used for the time-of-flight estimation
with the hybrid method [see (6)] is indeed the same as the duration \( (\Delta f)^{-1} \) of the chirp signal after pulse compression.

The main difference between the two methods is the filtering process. TDS relies on the frequency filtering of a signal through a sliding window. To achieve the proper rejection of the adjacent parts with a delay \( \Delta t_w \), the sliding window is reduced by as much as one-third (see [10], Eq.(24). In the hybrid method, the frequency selection is straightforward because the transmitting signals' duration is limited, avoiding the problems associated with the rejection of out-of-band interferences.

III. EXAMPLE OF CALIBRATION

A. Description of the experiment

This section presents an example of the application of the hybrid method. The notation used in this section is summarized in Table I and the Appendix.

1) Geometry of the setup

The task was to measure the receiving sensitivity and directivity of the 64 elements of a linear array. The total length of the array is \( L \approx 1.9 \) m. The actual size of the active face of each element is \( l_a \times l_b = 28 \times 7 \) mm. The frequency range is 80 to 130 kHz.

The measurements were conducted in a large tank that is part of the naval facilities in Brest. The tank is 80 m long, and 10 m wide, with a water column height of 8.4 m (Fig. 1).

![Fig. 1. Longitudinal section of the tank: layout of the elements with the receiving array in the horizontal configuration.](image)

Because of the particular context of this calibration, the sources are elements of another array driven by a complete transmitting setup (a 48-channel signal synthesizer and power amplifiers). This array is hung vertically, at the mid-width of one end of the tank. The upper side of this array is at \( h_r \approx 2.80 \) m depth under the surface. A single channel is active during each ping. Only five channels are used as sources. The geometric distribution of these sources \( \{ T_j \} \) within the array is given in Table 1.
Table 1. Relative vertical location of sources $T_j$ in the transmitting array ($T_1$ as reference, $T_5$ deeper than $T_1$)

<table>
<thead>
<tr>
<th>Source ($j$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta h_j$ (cm)</td>
<td>0</td>
<td>16.4</td>
<td>34.0</td>
<td>52.9</td>
<td>70.5</td>
</tr>
</tbody>
</table>

Increasing the number of sources is not mandatory. However, the use of a set of sources with well-defined relative locations brings two main advantages: 1) it improves the robustness of the fitting process for determining the less well-known geometric parameters; and 2) the sensitivities measured with the different sources can be averaged to improve the accuracy of the calibration.

The receiving array is deployed from a deck across the tank. The distance from the source is $d \approx 32$ m. The suspension of the array includes a vertical axis of rotation. The mobile carriage that supports this axis is adjusted laterally on the deck so that the receiving array is located as much as possible in the boresight of the sources. Two distinct configurations are used to fasten the receiving array to the axis: horizontal and vertical positioning of the array. Only the horizontal configuration is addressed here (Figs. 1 and 2) to illustrate the presented method. With this setting, the directivities of the elements are measured in the principal plane that contains the array. Because of the limited length of the connecting cable, the receiving array was not immersed deeper than $h^R = 1.8$ m. Hence, there is an altitude difference between the receiving array and the upper part of the transmitting array:

$$h_0 = h^T - h^R \approx 1 \text{ m}.$$ (20)

The source level of each transmitting channel ranges from 204 dB at 80 kHz, up to 208 dB at 130 kHz (re $1 \mu$Pa rms @ 1m). These levels are obtained with elements whose directivity is relatively narrow in the horizontal plane because the length of the source elements (60 mm) is oriented along that plane. Depending on the frequency, the horizontal aperture of the transmitted beams (at -3 dB) varies between 16° (at 80 kHz) and 10° (at 130 kHz). The half-length of the receiving array being about 1 m, the lateral misalignment with respect to the boresight of the sources remains smaller than 2° for the outer elements because of the large distance $d$ between the source and the receivers. Consequently, even though a slight level correction could be required in the calibration process, the studied array lies always plainly in the main lobe of the sources. In the perpendicular direction, the width of the source modules is only 12 mm. Hence, the vertical aperture counts as tens of degrees ($> 30^\circ$ at -3 dB). At the receiving side, the width of the element is still smaller ($l_b = 7$ mm). The difference in height between the receiving array and the deepest source ($T_5$) reaches $h^R - h^R = 1.7$ m. At 32-m horizontal distance, this shift yields an angular misalignment of 3°. This is negligible compared with the vertical apertures, both at transmit and at receive.
Fig. 2. Measurement of the directivity in azimuth. Sign convention: \(u_i > 0\), \(\theta < 0\) and \(\psi_i > 0\) in this figure.

Note that the source is viewed from the \(i^{th}\) element with an angle \(|\psi_i|\) that differs from the array orientation angle \(|\theta|\) (parallax).

One considers two left-hand reference frames. One frame is still; the other is attached to the receiving array. The axis of rotation defines the \(z\)-axis that is common for both frames by pointing downward. The array is mounted so that it is perpendicular to this axis. The origin is the same for both frames, so that the \(z\)-coordinates of all the array elements are null. The sources are aligned vertically, i.e., parallel to the \(z\)-axis. The direction between this axis and the sources defines the \(y\)-axis of the fixed frame. The coordinates of the sources \(T_j\) read

\[
T_j = \begin{bmatrix} 0 & d \\ h_j & h_{0j} + \Delta h_j \end{bmatrix},
\]

where \(\Delta h_j\) are given in Table 1.

The array frame makes an angle \(\theta\) with the fixed frame (\(\theta < 0\) in Fig. 2). The array is not exactly centered on the rotation axis: \((\delta_x, \delta_y)\) denotes the coordinates of the array center in the array frame. The abscissa of the \(i^{th}\) element, referenced to the array center, is denoted \(u_i\). Hence, the coordinates of this element are, in the fixed frame,
\[
\begin{bmatrix}
  x_i \\
  y_i \\
  z_i 
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & -\sin \theta & 0 \\
  \sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  \delta_x + u_i \\
  \delta_y \\
  0
\end{bmatrix},
\]

(22)

where \( u_i=1.64 = (i - 32.5) \Delta u \); \( \Delta u \), the step between elements is 29.5 mm. The distance \( r_j(u,\theta) \) between the source \( T_j \) and the point of the array whose relative abscissa is \( u \) is

\[
r_j(u,\theta) = \sqrt{(d \cos \theta - \delta_y)^2 + (u + \delta_x - d \sin \theta)^2 + h_j^2}.
\]

(23)

The orientation of the array is given by a telemetry system; however, the origin is not well defined (it is manually set). The difference between the indexed angle \( \phi \) and the actual orientation \( \theta \) is denoted \( \delta_\theta \).

\[
\theta = \phi - \delta_\theta.
\]

(24)

During the measurement, a symmetrical 120°-wide sector is scanned with a step of \( \Delta \phi = 2^\circ \) (61 angular settings \( \phi_m = -60^\circ + (m-1)\Delta \phi \) with \( m = 1...61 \)).

In this experimental example, the less well-known parameters are \( \delta_\theta \), \( \delta_x \), \( \delta_y \), \( h_0 \), and \( d \); the parameters \( \phi_m \), \( \Delta h_j \) and \( u_i \) are well defined.

2) Parallax

Although the distance \( d \) is significantly larger than the array length \( L \) (0.5 \( L/d \approx 1/32 \)), the difference \( \Delta \theta_i \) between the inverse \( -\theta \) of the array orientation and the actual angle \( \psi_i \) from which the \( i \)th element views the sources is not negligible:

\[
\tan \psi_i = \frac{u_i + \delta_x - d \sin \theta}{d \cos \theta - \delta_y},
\]

(25)

so that

\[
\Delta \theta_i = \psi_i + \theta = \arctan \frac{(u_i + \delta_x) \cos \theta - \delta_y \sin \theta}{d - (u_i + \delta_x) \sin \theta - \delta_y \cos \theta} \approx \frac{(u_i + \delta_x) \cos \theta - \delta_y \sin \theta}{d}.
\]

(26)

The latter approximation is obtained by taking into account that \( d \) is much larger than \( \delta_x \), \( \delta_y \), and \( u_i \). Fig. 3 displays the parallax angle \( \Delta \theta_i \). This angle reaches the order of magnitude of the angular sampling interval \( (\Delta \phi = 2^\circ) \). Hence, the bias that is introduced by the parallax must be taken into account for determining the directivity pattern of each element.
Fig. 3. Difference $\Delta \theta_j = \psi_j + \theta$ between the view angles from the boresight of the elements to the source $(\psi_j)$, and from the boresight of the array to the source $(-\theta)$, as a function of the element abscissa $u_j$ and the array orientation $\theta$.

3) Difference between direct path and surface reflection

The receiving array and the sources $T_j$ are located at depth $h_R$ and $(h_R + h_j)$, respectively. The horizontal distance $d$ is much larger than $h_R$ and $h_j$. It is also much larger than the variations with the array orientation of the horizontal distance between each individual element and the sources. Hence, the travel difference $\Delta r_j$ between the direct path from the source $T_j$ and the indirect path after reflection from the free surface can be approximated for all of the receiving elements and all of the array orientations by:

$$\Delta r_j = 2d^{-1}h_R(h_R + h_j).$$

The shortest differences occur with the source closest to the surface $(T_1)$. Given $h_R \approx 1.8 \text{ m}$, $d \approx 32 \text{ m}$, and $h_1 \approx 1 \text{ m}$, the order of magnitude is $\Delta r_1 \approx 31 \text{ cm}$, which is equivalent to $\Delta \tau_w \approx 210 \text{ \mu s}$. This delay corresponds to less than 17 periods at a frequency of 80 kHz (i.e., the lower bound of the bandwidth of interest). This small margin is, furthermore, reduced because of the increase that the transfer functions, both at transmit and at receive, induce in the recorded signals. On the other hand, the distances between the sources and most of the receiving elements vary much more than the delays (27) during the rotation of the array. It confirms that a rigorous determination of the selecting windows is called for in such a configuration.

4) Signals

A set of $n = 51$ synthesized signals was used, as described in Section II-A, all centered with a null value in the transmit window. The frequency range of 80 to 130 kHz was sampled with the step $\delta f = 1 \text{ kHz}$. The synthesized signals $e_k$ were gated sinusoids. To avoid driving the power amplifiers with steep transients, these signals were designed with an integer number of periods, beginning and
ending at the null transition. In keeping the same number periods, \( n_p \), for all the signals, the relative bandwidth is constant:

\[
e_k(t) = \text{rect}\left(\frac{f_k t}{n_p}\right) \sin\left(2\pi f_k t\right) \quad \text{with} \quad f_k = 80, 81, \ldots, 130 \text{kHz} \quad \text{and} \quad n_p = 12.
\] (28)

Note that Eq.(28) departs from (1) because the envelopes \( a_k(t) \) depend on the signals: the duration \( \Delta t_e \) of the signals varies between 92 and 150 \( \mu \text{s} \); however, it does not introduce any change in the central part of the sum (5). Because the scanned bandwidth is \( \Delta f = 50 \text{kHz} \), there is the order of magnitude \( \Delta f / \Delta t_e \approx 6 \), which is much smaller than \( n = 51 \). Hence, condition (4) holds, and the sum (5) can be approximated by (6). In addition, because the frequency sampling was sufficient, the contribution of the outer part of the signals in the sum is not significant compared to the central peak of the sinc function (Fig. 4).

The width of the peak is around \( (\Delta f)^{-1} = 20 \mu \text{s} \). This value is significantly smaller than the duration of the shortest synthesized signal \( \Delta t_e = 92 \mu \text{s} \) at \( f = 130 \text{kHz} \). Moreover, this peak width is much smaller than the smallest estimated delay \( \Delta t_w = 210 \mu \text{s} \) (27) between the arrivals of the direct signal and of the surface reflection. Thus, there is no risk of any confusion with the peak associated with the indirect path.

5) **Hydrophone location**

A calibrated, quasi-omnidirectional hydrophone (TC4034, Reson A/S, Slangerup, Denmark [8]) is used as a reference. It allows the receiving element responses to be derived without knowing the precise characteristics of the sources. Because of the directivity of the sources, the hydrophone must be located close to the receiving array. However, the location of the hydrophone must be also chosen to avoid the interference with echoes diffracted by the array structure. In addition, the direct signal recorded by the hydrophone must not be embedded in the surface-reflected echo. Practically, it was convenient to fasten the hydrophone to the vertical axis of rotation, directly above the central part of the receiving array (level with the front face). The vertical distance is about 30 cm. This setting reduces the time window available for the hydrophone \( \Delta t_w \approx 175 \mu \text{s} \) because it is closer to the
surface. However, this restriction is not of concern because the bandwidth of the hydrophone is quite large (more that 500 kHz), and by noting that the largest duration of the transmitted signal is 120 µs. When the array is oriented at large angles off the direction of the sources, there could be a potential corruption of the reference signals because of the diffraction coming from the front end of the array (113 µs delay at $\theta = 60^\circ$). Consequently, the reference signals are acquired with the orientation of the receiving array close to the boresight. Hence, the first possible echoes that can be diffracted by the receiving array come from the central part of the array, so that the path difference between the direct signals and the first diffracted signals is also about 30 cm [i.e., the same order of magnitude as (27)].

Conversely, the waves that are diffracted by the hydrophone towards the array do not hinder the signals received by the elements. These diffracted waves are likely to remain at a low level because of the small size of the hydrophone, and the impinging direction is well out of the transverse directivity pattern of the array elements.

**B. Data Processing**

1) **Initial Search for Wavefronts**

In this experiment, the number of transmit cycles is equal to 15,555 pings ($= 61$ angles $\times 5$ sources $\times 51$ frequencies), which produces 995,520 recorded signals $s_{i,\theta,j,\omega}(t)$ (because there are 64 receiving elements). The source index $j$ is omitted in this part because the sets of data obtained with the different sources are processed independent of each other. The first step in the data processing consists of computing the sums

$$
\mathcal{E}_{i,\theta}(t) = n^{-1} \sum_{\omega} s_{i,\theta,\omega}(t).
$$

The envelope $E_{i,\theta}(t)$ of these sums is then derived (i.e., magnitude of the analytical signal).

The top portion of Fig. 5 exhibits the envelopes obtained with the source $1_T$, at a few orientation angles $\theta$. The wavefront corresponding to the direct path followed by the first multiple reflected by the free surface is clearly delineated. The algorithm that detects the instants of arrival is designed to scrutinize the different orientations of the array successively. The process is initialized by hand with an estimate of the delays $t_{i,\phi}^{(e)}$ at the starting position $\phi = -60^\circ$. For any given orientation $\phi_m$, the instant $t_{i,\phi_m}$ of the maximum value of the envelope is searched for each receiving element (index $i$) within a window $\Delta t$ that is centered on the initial guess $t_{i,\phi_m}^{(e)}$:

$$
E_{i,\phi_m}(t_{i,\phi_m}) = \max \left( E_{i,\phi_m} \left( t \in \left[ t_{i,\phi_m}^{(e)} - \frac{\Delta t}{2}, t_{i,\phi_m}^{(e)} + \frac{\Delta t}{2} \right] \right) \right).
$$

A least-squares fitting with a quadric in $u_t$ is performed on the resulting delays $t_{i,\phi_m}$. These smoothed delays $t_{i,\phi_m}^{(s)}(u_t)$ are used to determine the centers of the search windows in the next iteration $m+1$, i.e.,

$$
t_{i,\phi_m}^{(e)} = t_{i,\phi_m}^{(s)}(u_t).
$$

The width of the search window (\(\Delta t = 100 \mu s\)) is set large enough to support the shift in the arrival time between two adjacent orientation angles (23 µs with a 1-m arm level and $\Delta \phi = 2^\circ$ angular interval). On the other hand, $\Delta t$ is small enough, i.e., less than $\Delta \omega = 210 \mu s$ [see (27)], that there is no risk of catching the second wavefront. The interest of using the fitted data $t_{i,\phi_m}^{(s)}$ is to improve the robustness of the iterative search from one array orientation to the next. A wrong time determination $t_{i,\phi_m}$ for an array element is thus less likely to propagate to the next orientation. Note
that the fitting with a quadric is justified because the distances from the elements to the sources are much larger than the lateral dimension of the array, i.e., the Fresnel approximation holds.

Fig. 5. (top) Envelope of the sum of the signals received at all frequencies (each horizontal line is normalized with the maximal value). (bottom) Location of the envelope maxima (blue dots) and the corresponding second-degree polynomial fitting (red line). (The abscissa axis is labeled with the sample index of the records. Sampling period is 1 µs)

Fig. 6 displays the results eventually found and expressed in terms of the distances \( r_{i,\theta} \) between the sources and the elements (assuming a constant sound speed equal to 1490 m/s). Although the process is robust enough to avoid confusing the arrivals from the direct path with latter interferences, fluctuations can be noticed that the geometry of the setup does not justify. As mentioned in Section II-B, one takes advantage of the model (23) to refine the less well-known geometric parameters and eventually to derive more-accurate times of flight. The goal is to ensure reliable settings of the processing windows for all the array orientations.
Fig. 6. Distance $r_{i,\theta}$ between the receiving elements and the source $T_1$, as a function of the element abscissa $u_i$ (referenced to the array center) and the array orientation $\theta$ (blue dots in Fig. 5).

2) **Determination of Geometric Parameters**

a) **Sequential determination**

With the estimated distances between the sources and the receiving array elements, it is possible to sequentially derive the less well-known parameters $\delta_\theta$, $\delta_x$, $\delta_y$, $h_0$, and $d$. This is indeed a case-specific approach to the search for the geometric parameters, but it is more intuitive than the direct least-squares fitting method. The details of this process are described in the Appendix. Only the outline is given here.

Looking at the set of distances between the elements and any individual source (Fig. 6), a saddle point can be observed near the origin of the plane $(u, \theta)$. While rotating around the vertical axis, each element of the receiving array follows a circular trajectory. The point $O'$ of the array whose abscissa is $u = -\delta_x$ describes the arc of a circle with the smallest radius, $\delta_y$. The coordinates of the saddle point are $(-\delta_x, 0)$: this corresponds to the configuration in which the axis of rotation, the point $O'$ and the source are aligned ($\theta = 0$). Looking at the projection of Fig. 6 in the plane $(r, \theta)$, the envelope of the curves $r_\theta(\theta)$ exhibits a maximum at the abscissa $\phi = \delta_\theta$ where the actual angle $\theta$ is null. From Fig. 7, one finds $\delta_\theta \approx -0.15^\circ$. Accordingly, $\delta_x$ can be estimated with the projection in the plane $(r, u)$ of Fig. 6 (see Fig. 8). The envelope of the chart $r_\theta(u)$ is maximal at the abscissa $u = -\delta_x$. Here, one estimates $\delta_x \approx 1.5$ cm.


Fig. 7. (left) Range from the source $T_1$ to the array elements as a function of the array orientation $\phi$. There is one curve $r_u(\phi)$ per array element. (right) Magnification of the central part of the figure on left.

Fig. 8. (left) Range from the source $T_1$ to the array elements as a function of the position of the element in the array. There is one curve $r_\theta(n)$ per array orientation. (right) Magnification of the central part on the central part of the figure on left.

From (23), the distance between $O'$ and any source $T_j$ can be approximated by

$$r_{O'j}(\theta) \approx c_{0j} - \delta_x \cos \theta,$$

where $c_{0j}$ does not depend on the orientation $\theta$. Once the parameters $\delta_x$ and $\delta_y$ are known, the evolution of $r_{O'j}$ as a function of the cosine of the array orientation can be thus analyzed. Fig. 9 shows that $r_{O'j}(\cos \theta)$ are indeed parallel lines, from which the opposite of the mean slope, $\delta_y \approx 33$ cm, is derived.
Fig. 9. Longitudinal offset \( \delta_y \) in the location of the array, derived as the mean slope of the distances
\( r_{O'j}(\cos \theta) = c_{0j} - \delta_y \cos \theta \) between the array point \( O' \) and the sources \( T_j \). Dashed line: linear regression.

The data corresponding to any of the five sources can be processed independently to derive the parameters \( \delta_y \), \( \delta_x \), and \( \delta_z \); the average values are used to improve the accuracy of these determinations. On the other hand, each source yields an average characteristic distance. Using the appropriate geometrical approximations, this set of distances can be processed to estimate the depth \( h_0 \approx 1.0 \text{ m} \) of the transmitting array, and finally to refine the horizontal mean distance \( d \approx 31.9 \text{ m} \) (see Appendix).

b) Direct model fitting

The model used with the direct, classical fitting technique is given by (23). The data \( r_{\exp} \) are the raw results displayed in Fig 6, i.e., without any smoothing process [blue dots in Fig. 5]. The minimization process reads:

\[
\frac{\partial}{\partial p} \left( C^2 \right) = 0 \quad \text{with} \quad C^2 = \sum_{i,j} \left( r_{\exp} - r_{\text{model},i,j} \right)^2,
\]

where \( p = \delta_y, \delta_x, \delta_z \), and \( h_0 \).

Implemented as an iterative process, the parameter \( d \) is corrected at each step to keep null the mean difference between the data and the model:

\[
\delta d = \left( r_{\exp} - r_{\text{model}} \right)_{i,j}.
\]

Initializing the searched parameters with the values obtained by the sequential approach, the convergence yields very close results: \( \delta_y = -0.20^\circ, \delta_x = 15 \text{ mm}, \delta_z = 319 \text{ mm}, \ h_0 = 1.01 \text{ m}, \) and \( d = 31.87 \text{ m} \).
The differences $\Delta r$ between the final model and the raw data are displayed in Fig. 10. The standard deviation $C$ is around 10 mm, i.e., the same order of magnitude as the wavelength, which is not negligible, considering that the signal length is only $12\lambda$. The largest differences occur around 30° on both sides of the main axis. These directions correspond to the transition areas between the main lobe and the first side lobe of the elementary directivity patterns (i.e., the theoretical transition occurs at $|\theta| = \sin(\lambda/l_a)$, e.g., $|\theta| = 32^\circ$ at 100 kHz with $l_a = 28$ mm).

Notice that all the geometric parameters are given in terms of the natural dimension, i.e., length. The model is actually based on time measurements and then converted to distances through the proportionality relation with the sound speed (i.e., 1490 m/s in seawater at ambient conditions considered here). The relative accuracy of this parameter is at best about 1/1000. Displaying the value of $d$ with four digits is therefore not significant in terms of the actual, absolute accuracy on the knowledge of this parameter; however, it is consistent with the accuracy that is obtained in terms of time to determine the selection windows. Regarding the sensitivity of the fitting process, the standard deviation $C$ increases by $\delta C = 0.1$ mm with the following deviation of the parameters: $0.2^\circ(\delta_\theta)$, 3 mm ($\delta_x$), 2 mm ($\delta_y$), and 4 cm ($h_0$). The influence of $d$ is indeed given by the increase in the standard deviation associated with the introduction of a nonzero mean, i.e., $\delta d \approx \sqrt{2C \times \delta C}$ ($\delta d = 1.4$ mm yields $\delta C = 0.1$ mm).

3) Final Windowing and Results

The process yields delays $t_{i,\theta,j}^{\text{mid}}$ that correspond approximately to the middle of the signals received by the $i^{th}$ element, from the source $T_j$, the orientation of the array being $\theta$. Given a particular frequency, the beginning of the reception can be retrieved by subtracting two contributions: the half-duration of the $n_p$-period synthesized signal and another small contribution that takes into account the signal spread caused by the angle between the incoming wavefront and the element face, the length of which is $l_a$:

$$t_{i,\theta,j,\omega}^{\text{init}} = t_{i,\theta,j}^{\text{mid}} - \left(\frac{n_p \cdot 2\pi}{\omega} + \frac{l_a}{c} \sin|\theta|\right)/2,$$

(34)
where \( n_p = 12 \), \( l_{a} = 28 \text{ mm} \), and \( c = 1490 \text{ m/s} \). The windows must not overlap the surface echoes.

Consequently, the upper limits \( t^\text{end} \) are defined with (27):

\[
\begin{align*}
   t^\text{end,ij,ω} = t^\text{init,ij,ω} + \Delta t' / c.
\end{align*}
\]

The windows are slightly shifted back to preserve a small safety margin \( \delta t \) that is commensurate to the mean signal period:

\[
\begin{align*}
   w_{i,θ,j,ω}(t) &= \text{rect}\left( \frac{t - t^\text{init} + t^\text{end} + \delta t}{2(t^\text{end} - t^\text{init})} \right).
\end{align*}
\]

All of the received signals \( s_{i,θ,j,ω} \) are processed within these windows as described in Section II-C (Fig. 11). The signals received by the hydrophone that is located close to the receiving array center undergo the same process except for the dependency with the angle \( θ \), which is not applicable here. The transfer functions accounting for propagation are computed with the distances \( r_{i,j} \). Note that because of the differential method, the effect of attenuation is negligible (\( α ≈ 34 \text{ dB/km} \) at 10.3 °C, and the path difference is less than 1 m). Knowing the sensitivity of the hydrophone, one obtains with (14) a set of receiving sensitivities. The values obtained from the five sources are averaged. The frequency and the angular sensitivity of each element (indexed \( i \)) is expressed in decibels as \( SH_i(ω,ψ) \):

\[
\begin{align*}
   \text{SH}_i(ω,ψ) &= \text{SH}_i(ω,ψ = 0) + D_i(ω,ψ)
\end{align*}
\]

with \( D_i(ω,0°) = 0 \text{ dB} \).

The average characteristics of the elements are presented in Figs. 12-13.

**Fig. 11.** (top) Example (with \( i = 16, \ θ = -16°, \ T_3, \ ω = 2π×100 \text{ kHz} \)) of signal clipping \( w_{i,θ,j,ω}(t) \),

and (bottom, real and imaginary parts) the corresponding integrand \( w_{i,θ,j,ω}(t)\tilde{s}_{i,θ,j,ω}(t)\exp(-j\omega t) \) in (8).
Fig. 12. Mean response $\langle SH_i(\omega, \psi) \rangle_i$ of the array elements (angular sampling interpolated with step $0.5^\circ$)

Fig. 13. Mean sensitivity $\langle SH_i(\omega) \rangle_i$ and directivity $\langle D_i(\omega, \psi) \rangle_i$ of the array elements
IV. Conclusion

The bounded geometry of experimental setups used to measure the directivity and receiving sensitivity of large array elements devoted to underwater applications restricts the applicability of free-field techniques, e.g., using tone bursts. The concern is indeed the possible interference between the direct transmitted wave and the waves reflected from the tank envelope. A straightforward solution to avoid such interferences is to transmit short pulses and then derive the transfer functions by spectral analysis. However, this approach can lead to poor signal-to-noise ratios. The TDS method overcomes this difficulty: the transmitted signal is a chirp that spreads the spectral energy in time; the unwanted interference is removed from the received records by a sliding filter. This technique was devised long ago because it could be implemented with the then-available analog network analyzers; in addition, it allows the resulting calibration to be collected quite rapidly, without post-processing recorded data. In any case, the correct timing of the analyzing window is mandatory. Although this setting can be managed when the calibrated transducer is compact, the tracking needed when rotating large arrays becomes more complicated. With digital techniques, the tracking method described in this paper can be used exactly in the same way with TDS, by compressing the received chirp signals. However, the main concern with TDS remains the quality of the filtering used to reject the interferences.

The proposed method is hybrid between the gated and the impulse methods: it uses sequences of properly phased harmonic signals that fit the width of the interference-free time windows. The resulting spectral spreading and the frequency resolution that can be achieved are strictly the same as with TDS. Centering the harmonic signals gives the same pulse compression capability at post-processing as with a chirp. The capital difference with TDS is the filtering process: with the hybrid method, interference is simply discarded by windowing the recorded signals. Consequently, whenever the digital equipment needed to synthesize and to record the signals is available, the hybrid method is preferred.

We presented an application case in which the precise boundaries of the direct signals needed to be delineated. Tracking these signals with the hybrid method is as simple and robust to implement as with the classical pulse compression scheme. In addition, fitting the obtained delays with a model based on the geometry of the setup enables retrieval of several less well-known offsets (e.g., mounting shifts and angles). It leads to delineation of with the time-windows that are free of the interfering signals with an improved accuracy. Note that the determination of the less well-known parameters could be also performed by implementing a sophisticated high-resolution method [5], but at the expense of complexity.

The reciprocal task consisting of the characterization of a multi-element transmitter using a receiving hydrophone can be handled with the same technique used to characterize a receiving array. The only practical difference is that the sequences of signals must be fed successively to each transmitter element. Considering the equivalent bandwidth, number of elements, and geometry, the total number of signals to be recorded by a reference hydrophone is the same in both cases. The post-processing of the received signals is also similar.
NOMENCLATURE

Subscripts and index

\( i \) Receiving array element
\( j \) Source
\( k \) Synthesized signal
\( m \) Orientation of the receiving array

For the sake of clarity, the indexed notation \( X_{i,j,k} \) is used instead of \( X_{i,m,j,k} \). This notation refers to a quantity related to the \( i \)th array element, the orientation setting \( \theta_m \) of the array, the source \( T_j \), and the synthesized signal with central pulsation \( \omega_k = 2\pi f_k \). For example, \( w_{i,j,k}(t) \) is the time window used to select the direct part of the received signal \( s_{i,j,k}(t) \); \( r_{i,j} \) is the distance between the source \( T_j \) and the \( i \)th element of the array whose orientation is \( \theta_i \) and \( t_{i,j} \) is the time of flight (direct path).

Geometric parameters

\( L \) Length of the receiving array
\( \Delta u \) Interelement spacing
\( l_a \times l_b \) Size of a single receiving element
\( h_{R}, h_{T} \) Depth below the surface (receiving array, upper part of the transmitting array)
\( h_0 \) Vertical difference between the upper part of the transmitting array and the receiving array
\( h_j \) Vertical difference between the source \( T_j \) and the receiving array.
\( \Delta h_j = h_j - h_0 \) Relative depth of the sources.
\( u_i \) Abscissa of the \( i \)th element relative to the array center
\( \delta_x, \delta_y \) Horizontal coordinates of the array center (offset relative to the axis of rotation)
\( \theta \) Orientation of the receiving array
\( \phi \) Measured orientation of the receiving array
\( \delta_\theta = \phi - \theta \) Mounting offset of the receiving array
\( \psi_i \) View angle from the \( i \)th element to the sources
\( \Delta \theta_i = \psi_i + \theta \) Parallax for the \( i \)th element with the orientation \( \theta \) of the array
\( d \) Horizontal distance between the sources and the axis of rotation
\( d_{i,\theta} \) Horizontal distance between the sources and the \( i \)th receiving element, with the orientation \( \theta \) of the array
SEQUENTIAL DETERMINATION OF GEOMETRIC PARAMETERS

In the sequential analysis, one processes the smoothed distances \( r_{\theta,j}(u) \) that are used in the initial search for the distances (red curves in Fig. 5). They are obtained by least-squares fitting with a quadratic of the sets \( r_{\theta,j} \). The abscissa \( u_{i} \) and the parameters \( \delta_{x}, \delta_{y}, \) and \( h_{j} \) are altogether much smaller than the distance \( d \), so that the development of (23) can be made explicit:

\[
 r_{j}(u, \theta) \approx d - \left( \delta_{x} \cos \theta + (u + \delta_{x}) \sin \theta \right) + \left( \frac{\delta_{x} \sin \theta - (u + \delta_{x}) \cos \theta}{2d} \right)^{2} + h_{j}^{2}.
\]  

(38)

This is the Fresnel approximation that develops the distances \( r \) as a quadratic form in \( u \):

\[
 r_{j}(u, \theta) \approx au^{2} + bu + c.
\]  

(39)

Coefficient \( a \) accounts for the curvature of the wavefront. Coefficient \( b \) is mainly dictated by the steering angle. Coefficient \( c \) is a close approximation of the distance between the sources and the center of the receiving array. Equating each term in (39) with (38) yields:

\[
\begin{align*}
 a &= \frac{\cos^{2} \theta}{2d} \\
 b &= -\sin \theta + \frac{\delta_{x} \cos \theta - \delta_{y} \sin \theta}{d} \cos \theta \\
 c &= d - \left( \delta_{x} \sin \theta + \delta_{y} \cos \theta \right) + \left( \frac{\delta_{x} \cos \theta - \delta_{y} \sin \theta}{2d} \right)^{2} + h_{j}^{2}
\end{align*}
\]  

(40)

The coefficients \( a \) and \( b \) are the same for all of the sources. They depend only on the orientation \( \theta \) of the array \( (a = a_{\theta} \) and \( b = b_{\theta} \). The coefficient \( c \) depends on both the sources depth and the array orientation \( c = c_{\theta,j} \).

a) Angular and lateral offsets

From the modeled distances given by (23),

\[
\frac{\partial r}{\partial u}_{\theta} = r^{-1}(u + \delta_{x} - d \sin \theta)
\]

\[
\frac{\partial r}{\partial \theta}_{u} = -r^{-1} d \cos \theta \left(u + \delta_{x} - \delta_{y} \tan \theta\right)
\]  

(41)

so that the saddle point (Fig. 6) is located at the point for which both gradients are null, i.e.,

\[
\frac{\partial r}{\partial \theta}_{u} = \frac{\partial r}{\partial u}_{\theta} = 0 \quad \text{at} \quad (u, \theta = \phi - \delta_{\theta}) = (-\delta_{x}, 0).
\]  

(42)

This point corresponds to the largest distance in the set containing the shortest ranges observed at each array orientation between the source and the array (Fig. 7-8).

b) Radial offset

The coefficient \( c \) provides the main contribution in computing the distances with (39): \( c \) in (40) is the distance between the source and the array center \( (u = 0) \). This point draws an arc of a circle when the array rotates. Using the first-order development with respect to \( \delta_{x,y} / d \), the dependency with \( \theta \)
reduces to:

\[ c_j(\theta) \approx c_{0j} - \left( \delta_x \cos \theta + \delta_y \sin \theta \right). \quad (43) \]

The length \( \delta_x \) being now available, and the curves defined using

\[ r_{0j}(\cos \theta) = c_j(\theta) + \delta_x \sin \theta \approx -\delta_x \cos \theta + c_{0j}. \quad (44) \]

can be drawn. Hence, \( \delta_y \) can be estimated with the slope of these curves (Fig. 9).

c) **Difference of altitude between the sources and the receiver**

One can define the following points associated with each source:

\[ \left( \Delta h_j, g_j \right) \text{ with } g_j = \left\{ c_{\theta,j} \right\}_\theta - \frac{(\Delta h_j)^2}{2d}. \quad (45) \]

\( \left\{ c_{\theta,j} \right\}_\theta \) are the averages over all array orientations of the coefficients \( c \), computed for each source; \( \Delta h_j \) are the accurately known relative positions of the sources in the transmitting array (Table 1); \( d \approx 32 \text{ m} \) is an approximate value for the horizontal distance between the sources and the axis of rotation. From (40), there is also

\[ g_j = \frac{h_0}{d} \Delta h_j + \text{const.}, \quad (46) \]

so that the interval \( h_0 \) can be derived from the slope \( d^{-1}h_0 \) of the line that fits the points (45). A graph (not presented here) shows that these points are indeed properly aligned, leading to \( h_0 \approx 1.0 \text{ m} \).

d) **Horizontal distance between sources and axis of rotation**

A final average is performed to derive a value of \( d \) with an improved accuracy. It consists of unfolding the expression of \( c \) in (40)

\[ g_j(\theta) = c_j(\theta) + \delta_x \cos \theta + \delta_y \sin \theta \]

\[ - \frac{(\delta_x \cos \theta - \delta_y \sin \theta)^2 + h_j^2}{2d} \approx d, \quad (47) \]

so as to derive the mean value computed over all the sources and all the array orientation angles: \( d \approx \left\langle g_j(\theta) \right\rangle_{\theta,j} \approx 31.9 \text{ m} \).

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