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Short communication

An new algorithm for calculating the exact Dolph-Chebyshev shading coefficients

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Résumé. — On commence par recenser les principales techniques de calcul des coefficients d'apodisation utilisant la méthode de Dolph-Chebyshev. On présente ensuite une nouvelle formulation exacte du calcul qui conduit à un algorithme rapide implanté sur un PC donnant des résultats précis même pour des réseaux de grande taille.

Abstract. — The main techniques for calculating the shading amplitude coefficients of linear arrays with the Dolph-Chebyshev method are first rewieved. A simple, new, exact formulation is then presented that can be calculated by a fast algorithm on a PC and yields accurate results, even for very large arrays.

Introduction.

Underwater imaging leads to the use of linear arrays of transducers. It is well known that it is necessary to use shading patterns to reduce sidelobe level [1, 2]. Among these patterns, the Dolph technique [3] is an interessant one : weighting the amplitudes of a linear array of N identical, equally spaced elements, so that the resulting pattern yields a minimum beamwidth with an arbitrary given main-lobe/sidelobe level ratio (r). This method depends on certain properties of the Chebyshev polynomials T_n:

$$T_n(x) = \sum_{k=0}^{n:2} a_{n-2k}^{(n)} x^{n-2k} = \begin{cases} \cos\left(n \cos^{-1} \quad (x)\right) & |x| \leq 1, \\ \cosh\left(n \cosh^{-1} \quad (x)\right) & x \geq 1. \end{cases}$$

with $a_{n-2k}^{(n)} = (-1)^k 2^{n-2k-1} n \dfrac{(n-k-1)!}{k!\,(n-2k)!}$.

("n:2" means the arithmetical division of n by 2, and the symbol ":" will keep this meaning in all subsequent integer expressions).

It can be shown that the proper N shading coefficients A_i (numbered symetrically from $i = -N$: 2 to N:2, excluding $i = 0$ when N is even) are found by solving the polynomial equation :

$$T_{N-1}(x_0 \cos\alpha) = \sum_{k=0}^{(N-1):2} A_{(N:2)-k} \cos([N-1-2k]\,\alpha), \quad (A_i = A_{-i})$$

where $x_0 (> 1)$ is deduced from $T_{N-1}(x_0) = r$.

Several expressions have been proposed to calculate these shading coefficients. The first (Barbiere [4], Stegen [5], Brown [6], Salzer [7]) were all based on finite series of terms of alternating sign. A representative expression of this group may be written with our notations :

$$A_{(N:2)-k} = (N-1)\sum_{i=0}^{k}(-1)^i \frac{(N-2-i)!}{i!\,(N-1-i-k)!\,(k-i)!} x_0^{N-1-2i}. \tag{1}$$

The main problem encountered with these formulations is that the values of each term in the series increase drastically with the number of elements. It yields a loss of significant figures during the numerical process, and the final results begin to diverge as soon as the antenna is made up with more than about 24 elements (double precision computation). The same kind of problem limits the extensive application of other methods based on Fourier transform (Diderich [8], Nuttal [9] and matrix (Balakrishnan *et al.* [10], Zielinski [11]) formulations. Van der Maas [12] and Drane [13] have given approximate solutions for large arrays which overcome this difficulty. These authors reorder the series so that they contain only positive terms. Van de Maas originally reaches this formulation by using a power series of the new variable $(x_0^2-1)/x_0^2$. Bresler [14] improves this technique, and gives an exact solution in the form of recursive nested products. The comparative study of Burns *et al.* [15] shows clearly that this last formulation leads to the fastest algorithm and the most accurate results.

In this paper, we present a similar recursive method based on another development. This algorithm is valid even for a large array (more than 100 elements). The numerical results are very accurate and can be obtained by fast single-precision computation on a personal computer.

The recursive algorithm.

The x_0 solution of the equation $T_{N-1}(x_0) = r$ can be written :

$$x_0 = \frac{1}{2}(b^{1/(N-1)} + b^{-1/(N-1)}), \text{ with } b = r + (r^2-1)^{1/2},$$

and it can be noted that the larger N is, the closer to unity is x_0. The expression (1) is difficult to compute because the limited number of significant digits leads to the loss of the difference between large powers of x_0 and unity. For this reason, we introduce the new variable $X = x_0^2-1$, noting that the increment in successive powers of x_0 is 2. Thus, we obtain the following expression :

$$A_{(N:2)-k} = x_0^{N-1-2k}\frac{(k-1)!}{(N-1-k)!}\sum_{i=1}^{k} X^i \frac{(N-2-k+i)!}{i!\,(i-1)!\,(k-i)!} \tag{2}$$

When N becomes large, it is no longer convenient to compute this sum straightforwardly. Rather, we save accuracy and computation time in expressing (2) as the nested product :

$$A_{(N:2)-k} = x_0^{N-1-2k} X \left[1 + X\frac{(k-1)(N-k)}{2\;\;1}\left\{1+ \dots \right.\right. \tag{3}$$

$$\left.\left.\left\{1 + X\frac{(k-i+1)(N-2-k+i)}{i\;\;.\;\;(i-1)}\left(1+\dots(1+X\frac{1.(N-2)}{k.(k-1)})\dots\right)\right\}\dots\right\}\right]$$

The expression (3) involves only sums and products of well-conditioned values, easy to program and fast to compute, even with small desktop machines, using only 16-bit real numbers. This nested product formulation is an alternative solution to Bresler's [14].

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