Frontiers of Nonlinear Acoustics: Proceedings of 12th ISNA
edited by M.F. Hamilton and D.T. Blackstock
Elsevier Science Publishers Ltd, London, 1990

OPTIMIZATION OF A PARAMETRIC TRANSMITTING ARRAY: THEORETICAL EXPRESSION OF THE SECONDARY FIELD GENERATED BY FINITE-AMPLITUDE NARROW PRIMARY BEAMS, USING A FOURIER ANALYSIS. NUMERICAL CHARTS AT 15 kHz

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ABSTRACT

This study is based on a Fourier analysis [4] and gives the paraxial solution of the secondary field generated by a parametric transmitting array at any distance of the antenna. The primary beams are supposed to be narrow and the saturation effects are taken into account using numerical extra-attenuation laws from a one-dimension model. Some numerical results are given concerning the farfield of circular pistons. These curves exhibit the optimal specifications (maximal secondary level vs. secondary beamwidth) that may be achieved with a difference frequency equal to 15 kHz and a given directivity. It shows that choosing the primary central frequency and the primary acoustic level is very critical. A large antenna (1m x 1.4m) is built in our laboratory to achieve an experimental confrontation.

INTRODUCTION

The power efficiency of a parametric transmitter is very low and it is known that increasing the primary acoustic level induces losses of parametric gains. The saturation effect causes a relative shortening of the source line which reduces the parametric directivity, the secondary level does not fully benefit from the increased power fed to the antenna. So, the directivity and intensity of the difference-frequency wave (DFW) are antagonist qualities. These performances depend upon three main factors: the central primary frequency, the source level and the size of the antenna. Many works have been done on this issue, based on a model (Moffett and Mellen [1]) that introduced a Taper function which reflects the distortion affecting the couple of primary waves. Extensive results for engineering purpose have been provided in Ref.2. The main difficulty encountered with the conventional approach arises from the complexity of integrating the source terms. It leads to use certain geometric assumptions and the secondary nearfield is very difficult to obtain. An efficient alternative method is to use a formalism based on a spatial Fourier analysis (Alais [3]). Many problems of the nonlinear acoustics can be reformulated in this way, including saturation limited propagation [4].

In the present paper, we take advantage of this formalism to derive the parametric field generated by finite-amplitude primary waves, using in other respects the following main hypothesis: narrow primary beams; preponderant role of the two primary beams to generate the DFW; nonlinear damping of these primary waves deduced from a one-dimension model. The equations used to compute numerical results have not been set with dimensionless parameters. It allows to use actual coefficients of viscous absorption in sea water. Thus, we present some curves concerning the farfield of circular pistons, with a 15 kHz difference frequency. These results have been used to design a large parametric antenna which is now being built.

THEORY

The transducer of a parametric transmitter is driven with two primary frequencies, but owing to the nonlinear interactions, numerous waves at the combination frequencies are generated during the propagation. Within the second order approximation, the global acoustic field, expressed in terms of the velocity potential ϕ, follows the classical wave equation: $\Box'\phi = S(\phi)$, in which S is the quadratic source term which reflects binary interactions between waves and $\Box'$ denotes the operator of propagation in absorbing media. There is no explicit way to solve it. However, it may be assumed to a certain extent that much of the energy of the parametric wave is provided by the interaction only of the primary waves. This idea is partly supported by the fact that the growth of the higher frequency components able to generate the DFW is checked by the linear absorption. The lower level of these components and their limited spatial extent, compared with the primary waves, lead to neglect these contributions. On the other hand, the acoustic levels of the waves at the low frequency combinations are supposed too weak to significantly alter the DFW. So then, the secondary field will be calculated with the wave equation: (indexes 1-2 refer to primary waves)

$$\Box'\phi_- = \frac{1}{c^2}\,\frac{\partial}{\partial t}\left\{\overrightarrow{\nabla\phi_1}^{*}\cdot\overrightarrow{\nabla\phi_2} + \frac{B}{2A}\,\frac{1}{c^2}\,\frac{\partial\phi_1^{*}}{\partial t}\,\frac{\partial\phi_2}{\partial t}\right\}. \qquad (1)$$

Let us now consider the linear propagation in a viscous fluid of a harmonic wave $\phi(r)\,exp(-j\omega t)$ radiating toward the average direction z, in the half space z>0 without source. In a plane Πz, this field can be written as the spatial inverse Fourier transform of the spectrum $A_z(f)$ related to this plane:

$$\phi(r) = \iint A_z(f)\, e^{2\pi j f.m}\, df, \quad (m \in \Pi_z,\ r = m + z) \qquad (2)$$

in which the real vectors **f** are the observed spatial frequencies in that plane. It can be shown that the following *inhomogeneous plane modes* [3] are solutions of the linear wave equation $\Box'\phi = 0$:

$$A_0\, exp\left[j[k.r-\omega t] - \frac{\alpha z}{\cos\theta}\right],\ \text{ with } k=\frac{\omega}{c},\ \mathbf{k}=\mathbf{k}_z+2\pi\mathbf{f},\ 2\pi f=k\sin\theta,\ \theta=(\mathbf{k},z) \qquad (3)$$

(α is the coefficient of linear absorption)

so that A_z is related to the spectrum A_0 in the reference plane Π_0 by:

$$A_z(f) = A_0(f)\, exp\left[-\frac{\alpha z}{\cos\theta} + jk_z z\right], \qquad (4)$$

and the field $\phi(\mathbf{r})exp(-j\omega t)$ (Eq.2) is seen as a sum of modes $(\mathbf{f},\omega)$ (Eq.3) weighted by the associated reference spectral values $A_0(\mathbf{f})$.

During nonlinear propagation, the space-time spectral components of the radiated fields change. Each harmonic beam can still be described with Eq.(2), but now with z-varying amplitudes of the reference spectra. The analysis of invariances in Eq.(1) shows that the interaction of two primary modes contributes to the growth of the difference mode (spatial and temporal frequencies). Due to the slow evolution of the amplitude of this mode when travelling a wavelength-like distance, Eq.(1) finally results in a first order differential equation:

$$\frac{dA_{0(-)}}{dz}(\mathbf{f}_-,z) = -\frac{\beta\omega_1\omega_2}{2c^3} A^*_{0(1)}(\mathbf{f}_1,z)\, A_{0(2)}(\mathbf{f}_2,z)\, e^{-\vartheta z}, \qquad (5)$$

with $\vartheta = (\alpha_1+\alpha_2-\alpha_-)-j(\Delta k_-)$, in which Δk_- is the gap between the source wave vector $\mathbf{k}_2-\mathbf{k}_1$ and the parametric wave vector $\mathbf{k}_-$. The z-dependance of the primary amplitudes forbids the analytical integration of Eq.(5). But its numerical evaluation can be significantly simplified with the following reasoning. The presence of phase shifts $(\Delta k)z$ between sources and created waves that appear when solving the second order wave equation with inhomogeneous plane modes shows that efficient nonlinear interactions occur only as far as the wave vectors of the contributing modes point to very close directions. According to the hypothesis that the angular apertures of the primary beams are very narrow, the combinations of harmonics generated by nonlinearity grow significantly only within the paraxial direction. It induces to evaluate the nonlinear damping of the primary waves with null Δks, i.e. within a one-dimension model. Thus, it enables to separate the variables $\mathbf{f}$ and z to express the primary spectral components:

$$A_{0(i)}(\mathbf{f},z) = u_i(z)\, A'_{0(i)}(\mathbf{f}), \quad (i=1,2) \qquad (6)$$

The most significant mechanism altering the parametric generation, i.e. the nonlinear decay of the source line, is described by means of the functions u(z). They are obtained by numerically solving a time Fourier expansion of the Burger's equation, in which the limit conditions are given by the velocity amplitude at the front face of the transducer. The distributions A' are dictated by the shape of the transducer. With a piston-like transducer, they identify with the spatial Fourier transform $\mathcal{A}(\mathbf{f})$ of the pupil function on the aperture of the transducer. The complete value of a secondary spectrum component is then obtained by integrating into the spatial frequency plane all the couples of modes that contribute to generate this mode. It leads finally to the pressure spectrum:

$$A^{(p)}_{0(-)}(\mathbf{f}_-,z) = \frac{j\rho_0\beta\omega_-}{2c} \iint \left[\mathcal{A}^*(\mathbf{f}-\mathbf{f}_-)\mathcal{A}(\mathbf{f}) \int_0^z u_1(z)u_2(z)\, e^{-\vartheta z} dz\right] d\mathbf{f}. \qquad (7)$$

These distributions are defined for every observation plane Πz. They may be understood as spectra returned to the z=0 plane, which correspond to fictive sources, entirely localized in the half-space z≤0. From such a spectrum, it is then possible to obtain the secondary field within Πz using the same transform as described with Eqs.(2-4). In the farfield of the primary beams, the Fraunhofer approximation provides the secondary field without this latter integration:

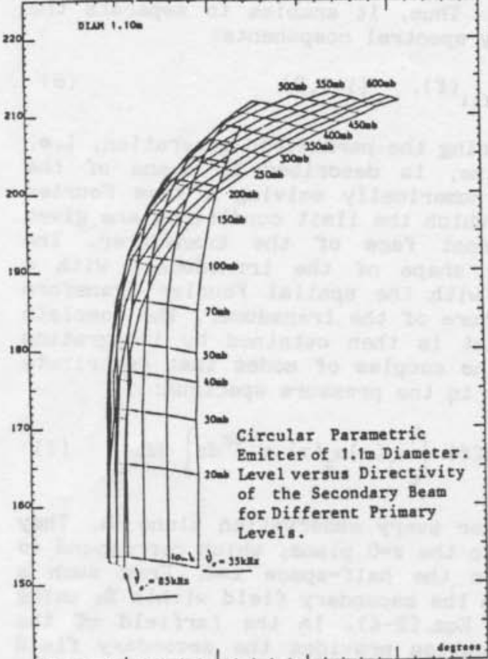
$$|P_-(\mathbf{r})| = \frac{1}{\lambda_- z} e^{-\alpha_- z} \left|A^{(p)}_{0(-)}\left(\frac{m}{r\lambda_-}\mathbf{n}\right)\right|. \qquad (8)$$

($\mathbf{n} = \mathbf{r}/r$ is a unit vector directed along the observed direction)

NUMERICAL RESULTS

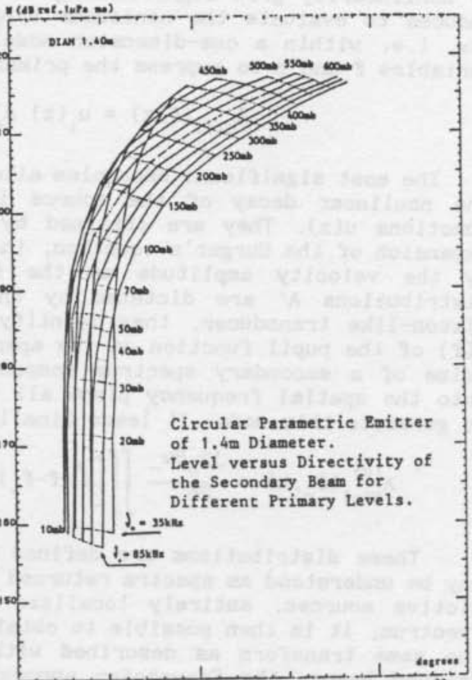
Although the parametric field may be computed with Eq.(2) at any distance z of the transmitter plane, it involves the preliminary computation of a complete mapping of each spectrum (Eq.7) related the observed plane. So, these complex calculations can only be undertaken for certain particular cases. In order to build the set of curves needed to design an optimized parametric transmitter, it is easier to consider only the farfield characteristics. With the Fraunhofer approximation, only few spectral values must be evaluated to obtain the on-axis level and the -3 dB beamwidth. Furthermore, a first approach may consider only circular transducers, and so takes advantage of the symmetry of revolution: the primary spectrum is given by a Fourier-Bessel transform.

We focus our interest on a parametric frequency of 15 kHz. The displayed curves exhibit the expected characteristics of the secondary farfield, i.e. the half-power width in abscissa and the acoustic level (dB ref. 1μPa *rms*| 1m) in ordinate. Figs. 1-2 show results with transducers whose diameters are respectively equal to 1.1 m and 1.3 m. Each curve is computed for a given primary central frequency (35 to 85 kHz by steps of 5 kHz), moving upward along a curve by increasing the transmitted power: the dashed lines indicate constant source levels expressed in terms of the pressure amplitudes $P_o = \rho_o c_o v_o$ (v_o is the velocity on the front face of the transducer for each primary signal). The effect of saturation is obvious as these curves are not straight vertical lines.



Circular Parametric Emitter of 1.1m Diameter. Level versus Directivity of the Secondary Beam for Different Primary Levels.

Figure 1



Circular Parametric Emitter of 1.4m Diameter. Level versus Directivity of the Secondary Beam for Different Primary Levels.

Figure 2

We are looking for optimized systems, which means for instance achieving the highest secondary level with a specified beamwidth. So, for a given diameter, these best performances are described with the envelope of the corresponding set of curves. At this point, a new and very practical result is seen: each optimized performance can only be reached by using a specific primary central frequency. Fig.3 is built with such envelopes for various diameters of transducers. A cross array of curves delineates the average primary frequencies required to obtain these results. The dashed line array indicates as previously the primary levels. So then, Fig.3 gives at a glance the farfield performances that can be expected with a 15 kHz parametric transmitter using a circular transducer, and the way to achieve them.

ANTENNA BEING BUILT

The specifications of the project undertaken in our laboratory can be summarized as follows: to generate a 15 kHz radiation with a half-power beamwidth less than 2°, and an acoustic level better than 205 dB. The

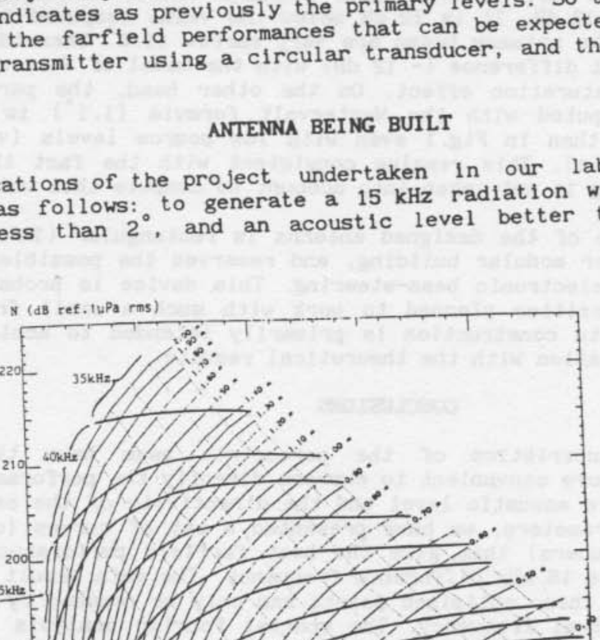


Figure 3 - Optimal secondary beams for diameters of 0.4 m to 2.0 m, with corresponding central frequencies an primary levels.

smallest diameter required to achieve such specifications is about 1.1 m, by using a 45 kHz central frequency with Po= 250 mb. The diameter of a linear transmitter should be greater than 3.1m to achieve the same resolution. The power output of the transducer is about $2 \times 210\ W/m^2$ (i.e. the total power is 400 W), which is not very high: the Goldberg's number in the one-dimension model is around 5 for each primary wave. The equivalent mean source level is 237 dB (for each primary beam), which gives a parametric gain of - 32 dB. It is 12 dB below the value computed with the Westervelt formula. The primary beams are very narrow (1.8° beamwidth), so that the on-axis level difference (- 12 dB) with the model of Westervelt is mainly due to the saturation effect. On the other hand, the parametric angular aperture computed with the Westervelt formula (1.1°) is always found to be smaller than in Fig.1 even with low source levels (vertical portions of the curves). This remains consistent with the fact that the primary wave spreading is not taken into account to compute this value.

The actual shape of the designed antenna is rectangular (1m x 1.4m), which allows an easier modular building, and reserves the possible future ability to bring an electronic beam-steering. This device is probably the only parametric transmitter planned to work with such a small frequency ratio, i.e. 3, and its construction is primarily intended to achieve the experimental confrontation with the theoretical results.

CONCLUSIONS

As the final characteristics of the parametric beam have the main significance, it is more convenient to express directly the performances in terms of the available acoustic level and the directivity of the secondary beam. Using these parameters, we have presented a set of curves (computed with circular transducers) that give the best farfield performances that can be expected with a 15 kHz difference frequency. The main result lies in the fact that any of these optimized points can only be reached by using a specific primary central frequency. The spatial Fourier analysis used to derive these numerical results is very convenient through approximations that it enables. But it has not been found realistic to undertake a huge exact numerical process involving the numerous space-time modes that propagate and interact. Instead, certain simplifications based on reasonable assumptions have been used. Thus, our results have to be compared with experimental data before completing other calculations with various difference frequencies and shapes of transducers. It must be noticed that the method allows also the computation of the nearfield.

REFERENCES

1. Moffett, M.B. and Mellen, R.H., Model for parametric acoustic sources. J. Acoust. Soc. Am., 1977, **61** 2, 325-337.

2. Scientific and Engineering Studies, Nonlinear Acoustics 1954 to 1983. Naval Underwater Systems Center, Newport (Rhode Island) and New London (Connecticut).

3. Alais, P., Effets de l'atténuation sur un rayonnement quelconque dans un milieu propagatif linéaire absorbant. C. R. Acad. Sc. Paris, 1976, Série A, **282**, 547-549.

4. Alais, P., Hennion, P.Y. and Cervenka, P., Etude par analyse de Fourier des phénomènes non linéaires affectant la propagation ultrasonore dans un fluide. L'émission paramétrique. L'auto-démodulation non linéaire. Rev. Phys. Appl., 1985, **20** 6, 285-297.