EXPERIMENTAL AND NUMERICAL STUDY OF NON-LINEAR INTERACTIONS IN THREE-DIMENSIONAL NOZZLE FLOW

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ABSTRACT: A prerequisite for aeroelastic stability investigations in internal and external aerodynamics is the understanding of the fluctuating forces acting on the structure. Unsteady transonic flows are complex because of mutual interactions between traveling acoustic waves, shock motion, and fluctuating turbulent boundary layers. This paper is concerned with the understanding of phenomena associated with pressure waves in non-uniform transonic flows and how they affect the unsteady pressure distribution as well as the shock motion. More precisely, the unsteady interaction of upstream propagating acoustic waves with a shock in a three dimensional transonic convergent-divergent nozzle is presented. Both numerical and experimental studies are carried out and compared with each other. Results show, at high frequency, the emergence of a high pressure amplification zone immediately downstream of the shock as an effect of the acoustic blockage theory. Furthermore, a systematic jump of the phase angle of the unsteady pressure perturbations is observed underneath the shock location. Interestingly, this phase shift varies both linearly with the perturbation frequency, and transversally with the strength of the shock. Numerical simulations also reveals a highly non linear behaviour of the shock motion through the appearance of a second sonic pocket during part of the unsteady cycle. In particular, the transversal propagation of the second sonic pocket is believed to correlate transversal and longitudinal acoustic modes within the channel.

1 BACKGROUND

Transonic flows about streamlined bodies are strongly affected, particularly near the shock location, by unsteady excitations. Experimental [1,2] and computational studies [3] have shown that the unsteady pressure distribution along the surface of an airfoil or a cascade blade in unsteady transonic flow exhibits a significant bulge near the shock location. Tijdeman and Seebass [4] reported that the unsteady pressure bulge and its phase variation resulted from non-linear interaction between the mean and unsteady flow. Beside, studies on choked flutter [5] showed that, in unsteady transonic flows around a single airfoil, the shock motion, and then the pressure distribution along the surface, can be critical regarding to the self-exciting oscillation of the airfoil. Especially, it was pointed out that the mean flow gradients are of high importance regarding the time response of the unsteady pressure distribution on the airfoil surface.

Further studies [6,7,8] suggested that this sharp rise in the unsteady pressure distribution was due to the near sonic flow conditions, and that the near-sonic velocity acts as a barrier they called acoustic blockage preventing acoustic disturbances from propagating upstream in a similar way to the shock in transonic flows. A two dimensional (2D) transonic convergent-divergent nozzle experimentally and numerically investigated by Bron et al. [9-11] was thereafter used as a model to investigate the unsteady shock boundary layer interaction and the non-linear acoustic blockage. Results showed that the amplification of the upstream propagating pressure disturbances downstream of the shock strongly depends on both the strength of the shock and the reduced frequency. Particularly, in the presence of a shock induced separation, a strong dependence on the reduced frequency was observed leading either to an amplification or an attenuation of the back pressure fluctuations. Furthermore, a shift in the phase angle distribution underneath the shock location was found to be linearly dependent on both the shock strength and the reduced frequency. A numerical investigation of the separated region revealed (Bron et al. [11]) that the separation point oscillates in phase and linearly with the shock motion. The reattachment
point, on the other hand, featured a non linear and out of phase behaviour compared to the shock oscillation. Surprisingly, the phase lag between the reattachment and the shock motions was found to be correlated to the reduced frequency.

2 INTRODUCTION AND OBJECTIVES

Following the previous research on the 2D nozzle performed by Bron [9-11], a three-dimensional (3D) nozzle test object was theoretically designed and numerically validated by Bron [12] to reproduce the structure of the radial pressure gradient usually encountered in turbomachinery flow. The design objectives were basically to create a bent shock configuration in which the point where the sonic line and the shock meet on the curved surface would be located in a low mean flow gradients region as sketched in Figure 1. The design procedure simply consisted in providing a transversal evolution to the mid-channel profile. The exact design parameters and evolution laws were presented in detail in Bron [13]. As a result, the 3D test object consists of a bump with varying thickness and length in the spanwise direction i.e. transversally to the mean flow direction. The leading edge of the bump was set as the origin of the reference system later used.

3 EXPERIMENTAL MODEL

Experiments have been performed in a conventional open loop wind tunnel facility. The air supply apparatus consists of 1MW skew compressor delivering a maximum mass flow of 4.7kg/s at 180°C. A condenser and cooling system then provide a flow at the working temperature of 30°C. The adjustment of different valves thereafter allows to control independently both the mass flow and the pressure level in the test section within the range of $M_a=0.6-0.8$ and $Re=\rho U a L/\mu=5.02-5.51 \times 10^6$ based on the distance between the first throat and the beginning of the bump, $L=0.33m$. The test section consists a 100x120mm rectangular channel (Figure 2) which is accessible from all four side walls for instrumentation and visualization purposes.

The investigated test object consists of a 3D bump described above inserted into a slot in the test section as shown in Figure 3. The bump is equipped with 350 pressure taps, which can be used either for steady state or high frequency pressure measurements. Fast response Kulite transducers were glued in protective pipes in order to move them easily between the different pressure holes. Although this
procedure has the advantage to only use a limited number of transducer, it however creates a pneumatic line, between the transducer and surface, which has to be dynamically calibrated.

Backpressure perturbations were introduced in the flow field by rotating an elliptical cam placed at x=630mm in the reference system of the bump. A DC motor was used to rotate the cam up to 15,000RPMs in order to generate pressure perturbations up to 500Hz. The rotating speed was monitored using an optical encoder located directly on the shaft of the motor. Rotating speed fluctuations and time drift were measured under ±0.024% in the worst case. Furthermore, a TTL pulse generated by the motor was used as a reference signal during unsteady pressure measurements.

Steady state pressure measurements have been performed using a 208-channels 'low speed' data acquisition system. Each pressure scanners feature a pressure range of ±100kPa relative to atmosphere with an accuracy of ±0.042% full scale. Taking into account the digital barometer, the overall accuracy for steady state pressure measurements is about ±43.5Pa. The sampling frequency and sampling time were respectively set to 10Hz and 200s in order to 'capture' the lowest frequencies.

Additionally, a 32-channels high frequency data acquisition and storage system was used for unsteady pressure measurements. Accounting for the resonance frequency of the capillarity pipes between the bump surface and the transducer, the sampling frequency was set to 8kHz with a low pass filter at 4kHz to avoid bias effects. A static calibration of all fast response transducers was performed prior and after the measurements in order to reduce the systematic error relative to the drift of the sensitivity and offset coefficients. A dynamic calibration was performed on all pressure taps in order to estimate the damping and time delay of propagating pressure waves through the capillarity tubes. The unsteady pressure measurements were thereafter corrected to account for the above estimated damping and phase-lag.

Steady state operating flow conditions were set up by adjusting the inlet total pressure, inlet total temperature, and outlet static pressure as summarized in Table 1. Unsteady operating flow conditions were thereafter estimated by measuring the change in back pressure between the extreme positions (vertical and horizontal) of the downstream rod and finally setting the averaged value in order to match the desired steady state operating condition.

The data reduction basically consisted in performing a Discrete Fourier Series Decomposition (DFSD) on the ensemble averaged signal obtained over several cycles of unsteady measurements, including the corrections for static and dynamic calibrations. The outlet pressure signal was thereafter used as a reference to calculate the pressure amplification factor and phase angle of the unsteady pressure distribution.

<table>
<thead>
<tr>
<th>Table 1: Experimental operating conditions during steady and unsteady pressure measurements</th>
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<tr>
<td><strong>Steady state OP</strong></td>
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<tr>
<td>Estimated accuracy</td>
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</table>

| **Unsteady OP** | Perturbation frequencies: 50, 100, 250, 500Hz | Perturbation Amplitude ±1.1kPa |
|-----------------|-----------------------------------------------|
| Vertical Rod    | 160.60 kPa | 303.3 K | 117.00 kPa | 0.637 | 3.59 kg/s |
| Horizontal Rod  | 159.46 kPa | 303.3 K | 114.80 kPa | 0.647 | 3.60 kg/s |
| Averaged        | 159.73 kPa | 303.3 K | 115.9 kPa |

4 NUMERICAL MODEL

Simulations were performed using a computational model [13] developed to simulate steady and unsteady, viscous and inviscid flows. The equations solved are the fully three-dimensional, unsteady, compressible, Reynolds averaged, Navier-Stokes (RANS) equations cast in the absolute frame where the laminar viscosity is assumed constant. The space discretization is based on a MUSCL finite volume formulation with vertex variable storage. The convective fluxes are evaluated using an upwind scheme based on Roe's approximate Riemann solver. The viscous terms are computed by a second order centered scheme. The turbulence closure problem is solved using Wilcox k-ω two equations linear model and fully
accounts for the effect of the boundary layer separation, which originates at the shock foot location. Compatibility relations are used to account for physical boundary conditions. The resulting semi discrete scheme is integrated in time using an explicit five steps Runge-Kutta time marching algorithm.

The fluid is modeled as a viscous perfect gas. The specific heat ratio is $\gamma = 1.4$, and the perfect gas constant equals $R = 287 \text{ J.kg}^{-1}.\text{K}^{-1}$. The laminar dynamic viscosity and the thermal conductivity are assumed constant and respectively equal $\mu = 1.81 \times 10^{-5} \text{ kg.m}^{-1}.\text{s}^{-1}$ and $\kappa = 2.54 \times 10^{-2} \text{ m.kg.K}^{-1}.\text{s}^{-1}$.

Inlet boundary conditions in the free stream were set accordingly to the experiments and are summarized in Table 2. A 7mm thick inlet boundary layer profil, computed over a simple rectangular duct calculation, was defined in the near wall regions. The outlet static pressure was adjusted, accordingly to the experiments, to $P_{\text{outlet}} = 116$ kPa.

### Table 2: Numerical operating conditions during steady and unsteady calculations

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{in}}$</th>
<th>$T_{\text{in}}$</th>
<th>$P_{\text{out}}$</th>
<th>$M_{\text{in}}$</th>
<th>$Q_{\text{in}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state OP</td>
<td>160 kPa</td>
<td>303 K</td>
<td>116 kPa</td>
<td>0.651</td>
<td>3.78 kg/s</td>
</tr>
<tr>
<td>Unsteady parameters</td>
<td>Perturbation frequency 500Hz</td>
<td>Perturbation Amplitude $\pm 2.32$ kPa</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

### 5 RESULTS AND ANALYSIS

#### 5.1 Steady State Results

The result of the 3D bump curvatures can be clearly observed on the isentropic Mach number distribution calculated from experimental pressure measurements in Figure 4. A transversal evolution of the mean flow gradients creates a high velocity region over a short distance on one side of the channel whereas a lower flow velocity region can be observed over a longer distance on the other side. At these operating conditions, a small sonic pocket has developed itself over the width of the channel, following the throat line location (dashed line in Figure 4) and creating a bent shock configuration. It is interesting to note the same shock structure with the oil visualization technique. Due to the strong deceleration through the shock, some paint stagnates and can be seen on the surface. However, as the strength of the shock varies throughout the width of the channel, the stagnating paint slightly vanishes in the low mean flow gradients region.

Similarly, Figure 5(right) presents a visualization of the sonic pocket obtained with steady state 3D RANS calculation. The agreement with the experiments is fairly good concerning the shock location and shape. On the other hand, the extent of the corner separation differs between the oil visualization and the numerical simulation. This difference is believed to originate the under prediction of the shock boundary layer interaction (SBLI) and side wall boundary layer thickening. However, although the simulation quantitatively under predicts the extent of the corner separation, a close comparison with the oil visualization reveals a quit similar 3D flow structure. As shown in Figure 5(left), the corner flow interaction can be analyzed using elementary singularities as defined by Legendre [14]. Particles from the main stream are attracted by the corner reattachment and impact on an isotropic node ($I_{\text{reat}}$) located on the side wall. From this location, flow particles migrate upstream in the reversed flow either through the saddle point (S) towards the focus of reattachment ($F_{\text{reat}}$) where they will spiral out and finally exit the recirculation, or directly to the focus of separation ($F_{\text{sep}}$) where they are immediately ejected into the main stream flow.

Although a fairly good agreement was achieved concerning the shock structure, an important difference was observed on the pressure distribution immediately downstream of the shock location. As seen in Figure 6, experimental results exhibit a much longer pressure relaxation zone than the numerical simulation, especially in the low mean flow gradient region. Once again, this difference is believed to originate the numerical under prediction of the SBLI, which in the experimental case leads to larger thickening and destabilization of the boundary layer.
4.2 Unsteady results

The pressure amplification factor calculated from unsteady pressure measurements has been plotted in Figures 7 and 8 for a perturbation frequency of 50Hz and 500Hz respectively. It is interesting to note that the region immediately downstream of the shock location features a high pressure amplification factor at high frequency and almost no amplification at all at low frequency. This observation results from the presence of a low wave length perturbation in a high velocity flow region and is in agreement with the acoustic blockage theory identified by Atassi [6]. Indeed, as back pressure perturbations propagate upstream in a high velocity flow region, their relative propagating speed decreases, and consequently, their wave length. By conservation of acoustic energy, the amplitude of the pressure fluctuations increases. This phenomenon is then more pronounced at higher frequency or in near sonic flow regions. Similar high pressure amplification factor (value about 3) can be observed on the 3D RANS calculation in Figure 9. The extent of the amplified zone is however shorter than in the experimental case as the numerical steady state solution under predicted the size of the pressure relaxation zone downstream of the shock (see Figure 6).

The evolution of the amplification factor through the width of the channel, observed for both experiments and simulations in Figures 7 to 9, is however not especially pronounced although a transversal pressure gradient is present. The flow velocity downstream of the shock appears quit constant through the width of the channel. Indeed, it seems that the transversal evolution of the shock strength is not important enough to create a significant variation of the flow velocity in the transversal direction.
Figure 7: Experimental unsteady pressure amplification at 50Hz. At y=20%(left), at y=50%(middle), at y=80%(right) of channel width.

Figure 8: Experimental unsteady pressure amplification at 500Hz. At y=20%(left), at y=50%(middle), at y=80%(right) of channel width.

Figure 9: Unsteady pressure amplification from 3D RANS simulation. At y=20%(left), at y=50%(middle), at y=80%(right) of channel width.

On the other hand, the transversal evolution of the phase angle jump observed underneath the shock location (illustrated in Figure 10 for the numerical calculation) does depend both on the shock strength and the frequency of the perturbations as observed in Figure 11. This result is in agreement with previous studies (Bron et al. [12,13]) on a 2D geometry for different shock strengths and perturbation frequencies. It is quite interesting to observe such transversal evolution of the phase shift through the shock. This result somehow confirms the idea, enounced by Ferrand [5] that mean flow gradients have rather an influence on the phase lag of traveling pressure perturbations than on their amplification. From an overall perspective, this phase shift, which seems to increase linearly with the perturbation frequency, is extremely important considering aerelastic stability predictions. Indeed, the imaginary part of the unsteady aerodynamic load on an airfoil is directly influenced by the value of this sudden phase change and the overall stability of the airfoil is therefore directly related to this value.

The analysis of the shock motion on the numerical simulation revealed a few interesting points. First of all the shock is oscillating as a “solid body”, that is to say there is almost no phase lag through the width of the channel. This observation reveals, on one hand, that the main flow is actually quit uniform downstream of the shock (the perturbations would propagate at different relative speed otherwise) and that the transversal pressure gradient does not affect the phase lag between the pressure perturbations and
the shock motion on the other hand. This last remark contrasts with the analytical study performed by Ferrand et al. [8] on the unsteady flow amplification produced by upstream or downstream disturbances. However, it should be noted that the analysis was performed using a quasi 1D hypothesis whereas the flow within the channel is highly 3D.

Another interesting observation was the appearance of a second sonic pocket during part of the unsteady cycle, while the shock is moving downstream from its most upstream position \((t=5/8T)\) in Figure 12. This behaviour is both due to the near sonic flow conditions at this location, and the steep front of the incoming pressure waves, which is magnified by the acoustic blockage effect and the reduced wave length of the perturbations. It is noteworthy that this second sonic pocket first appears at the location where the sonic line and the shock meet, i.e. in the low mean flow gradient region, and thereafter propagates transversally while colliding with the main sonic pocket. Clearly, the appearance of the second shock corresponds to a non linear behaviour and generates high harmonics in the unsteady pressure distribution downstream of the shock as observed in Figure 8 and 9. Furthermore, the extension of the second shock through the width of the channel also corresponds to a pressure wave propagation and therefore to the appearance of a transversal acoustic mode. As a result, longitudinal and transversal modes within the channel are being coupled by the appearance of this second sonic pocket. An acoustic decomposition, including streamwise, transversal and decaying modes would be interesting to verify this theory, and help deepen the analysis and quantify the interaction between upstream propagating perturbations and the nonuniform 3D flow field.

5 CONCLUSION

Steady state and unsteady pressure measurements have been performed, together with 3D RANS simulations, on a 3D convergent-divergent nozzle in order to investigate the interaction between upstream propagating acoustic waves in a non uniform transonic flow. A fairly good agreement was obtained between experimental and numerical steady state results, hereby validating the achievement of the test object design objectives. A fluctuating back pressure was thereafter applied in order to put the shock into an oscillation and study both the unsteady pressure distribution on the surface and the shock motion. Results showed, at high frequency, the emergence of a high pressure amplification zone immediately downstream of the shock as an effect of the acoustic blockage theory. Although the transversal evolution of the mean flow gradients created a variation of the shock strength through the width of the channel, it did not affect the transversal distribution of the pressure amplification nor the phase lag between pressure perturbations and the shock motion. On the other hand, it was observed a transversal evolution of a phase angle jump underneath the shock location. In agreement with previous
studies, this phase shift was found to linearly vary with the frequency of the pressure perturbations and increase proportionally with the strength of the shock. Numerical simulations also revealed a highly non linear behaviour of the shock motion through the appearance of a second sonic pocket during part of the unsteady cycle. In particular, the transversal propagation of the second sonic pocket is believed to correlate transversal and longitudinal acoustic modes within the channel.

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