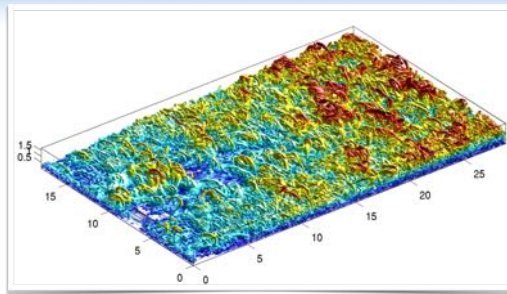


## Couches Limites Turbulentes

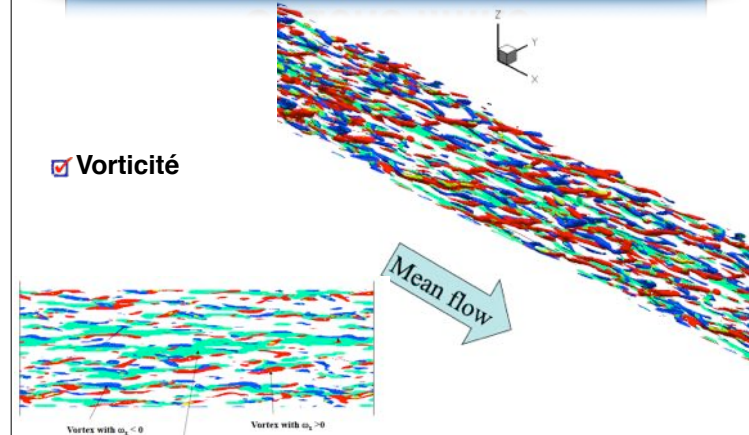


Juan A. Sillero

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## Couche limite

Vorticité



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## Couche limite

Effets de paroi

Pression : maintenir l'incompressibilité

Effet cinématique : assurer la condition d'imperméabilité à la paroi

Effet dynamique : effet echo du à la non localité de la pression

Viscosité :

dissipation dominante à la paroi

inhomogénéité normale à la paroi

mécanismes de redistribution de l'énergie fonctions de la distance à la paroi

Cisaillement :

condition de non glissement

gradient de champ moyen

Production d'énergie cinétique turbulente + anisotropie

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## Couche limite

Description: Longueurs caractéristiques

$$\delta_1 = \int_0^\delta \left(1 - \frac{\bar{u}(y)}{\bar{u}_\infty}\right) dy \quad \text{Epaisseur de déplacement}$$

$$\theta = \int_0^\delta \frac{\bar{u}(y)}{\bar{u}_\infty} \left(1 - \frac{\bar{u}(y)}{\bar{u}_\infty}\right) dy \quad \text{Epaisseur de quantité de mouvement}$$

$$\delta_3 = \int_0^\delta \frac{\bar{u}(y)}{\bar{u}_\infty} \left[1 - \left(\frac{\bar{u}(y)}{\bar{u}_\infty}\right)^2\right] dy \quad \text{Epaisseur d'énergie cinétique}$$

$$H = \delta_1/\theta, \quad H_{32} = \delta_3/\theta$$

$H \simeq 1.27$  Laminaire

$H \simeq 2.23$  Turbulent

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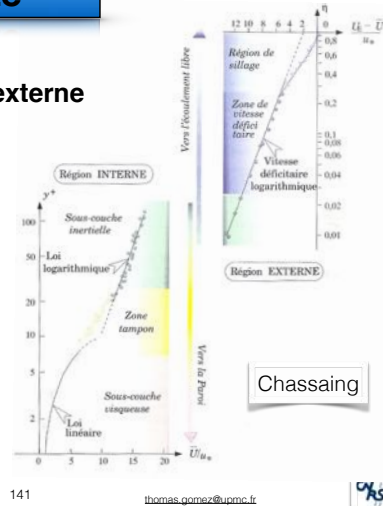
## Couche limite

### ☑ Région interne / Région externe

$$\tau_* \equiv \mu \left. \frac{d\bar{u}}{dy} \right|_{\text{wall}}$$

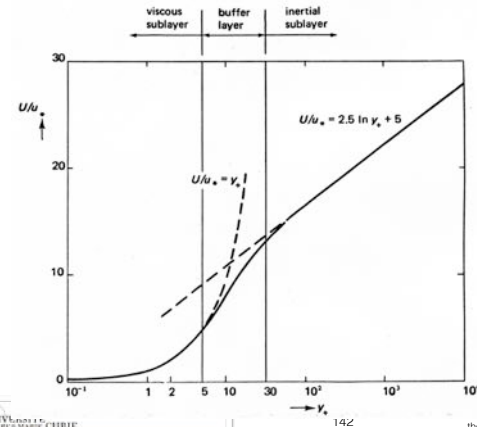
$$u_* \equiv \sqrt{\frac{\tau_*}{\rho}} = \sqrt{\nu \left. \frac{d\bar{u}}{dy} \right|_{\text{wall}}}$$

$$l_* \equiv \frac{\nu}{u_*} = \sqrt{\frac{\nu}{\left. \frac{d\bar{u}}{dy} \right|_{\text{wall}}}}$$



## Couche limite

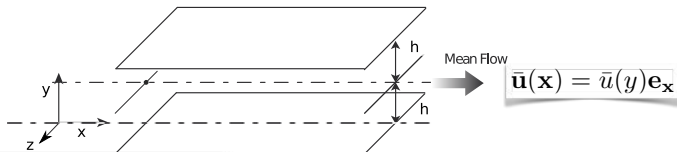
### ☑ Mean velocity profile



## Couche limite

### ☑ Canal plan : Equation pour le champ moyen

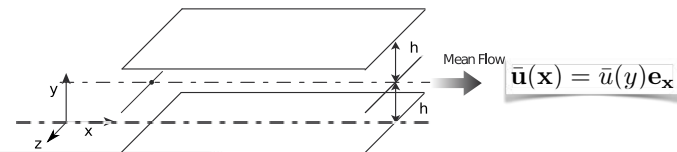
$$\begin{aligned} 0 &= -\frac{\partial \bar{p}}{\partial x} + \nu \frac{d^2 \bar{u}}{dy^2} - \frac{dR_{12}}{dy} = -\frac{\partial \bar{p}}{\partial x} + \frac{d}{dy} \left[ \nu \frac{d\bar{u}}{dy} - R_{12} \right] \\ 0 &= -\frac{\partial \bar{p}}{\partial y} - \frac{dR_{22}}{dy} = -\frac{\partial}{\partial y} [\bar{p} + R_{22}] \\ 0 &= -\frac{dR_{32}}{dy} \end{aligned}$$



## Couche limite

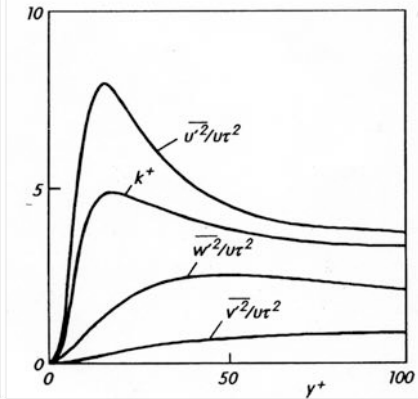
### ☑ Canal plan : Equation pour l'énergie cinétique

$$\begin{aligned} 0 &= -\frac{\partial}{\partial x} (\bar{p}\bar{u}) + \nu \frac{d^2 K}{dy^2} - \nu \left( \frac{d\bar{u}}{dy} \right)^2 - \frac{d}{dy} (\bar{u}R_{12}) + R_{12} \frac{d\bar{u}}{dy} \\ &= -\bar{u} \frac{\partial \bar{p}}{\partial x} + \frac{d}{dy} \left[ \nu \frac{dK}{dy} - \bar{u}R_{12} \right] - \frac{d\bar{u}}{dy} \left[ \nu \frac{d\bar{u}}{dy} - R_{12} \right] \end{aligned}$$



## Couche limite

### Canal plan : Distribution spatiale des fluctuations



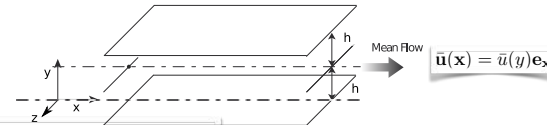
Schlichting et al.  
2000

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## Couche limite

### Canal plan : Equation pour les fluctuations

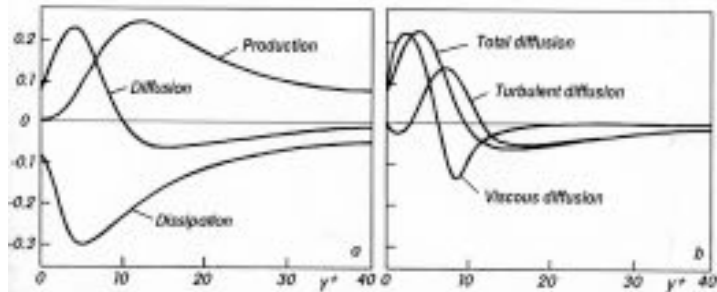
$\frac{D}{Dt}$	Production	Diffusion verticale	Pression	Dissipation
$0 =$	$-2R_{12} \frac{d\bar{u}}{dy}$	$+\frac{d}{dy} \left( -\overline{u'v'} + \nu \frac{d}{dy} R_{11} \right)$	$+\Pi_{11}$	$-\epsilon_{11}$
$0 =$		$\frac{d}{dy} \left( -v'(v'v' + 2p') + \nu \frac{d}{dy} R_{22} \right)$	$+\Pi_{22}$	$-\epsilon_{22}$
$0 =$		$\frac{d}{dy} \left( -v'w'w' + \nu \frac{d}{dy} R_{33} \right)$	$+\Pi_{33}$	$-\epsilon_{33}$
$0 =$	$-R_{22} \frac{d\bar{u}}{dy}$	$+\frac{d}{dy} \left( -\overline{u'v'v'} + \nu \frac{d}{dy} R_{12} \right)$	$+\Pi_{12}$	$-\epsilon_{12}$
$0 =$	$-R_{12} \frac{d\bar{u}}{dy}$	$+\frac{d}{dy} \left( \frac{1}{2} v'(u'u' + v'v' + w'w') - p'v' + \nu \frac{d}{dy} \mathcal{K} \right)$		$-\epsilon$



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## Couche limite

### Canal plan : 3 mécanismes

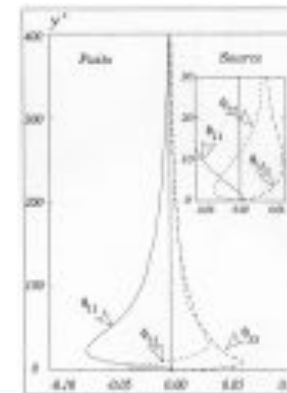


Schlichting

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## Couche limite

### Corrélation pression-déformation

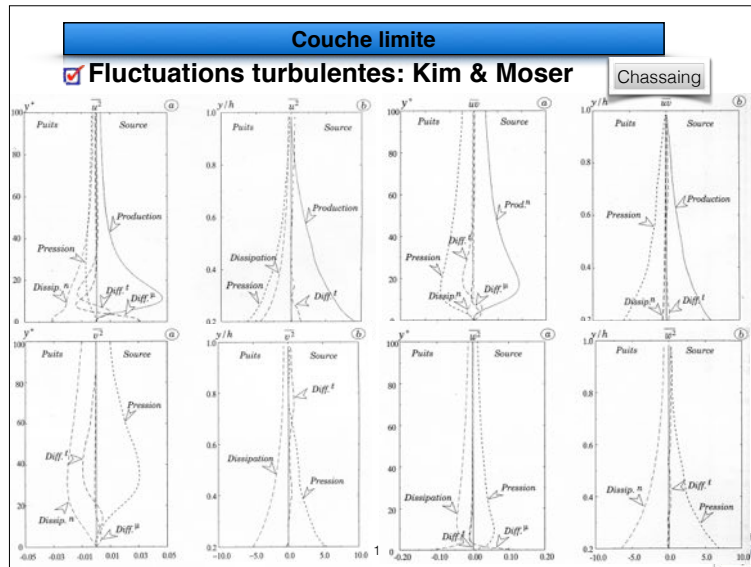


Chassaing

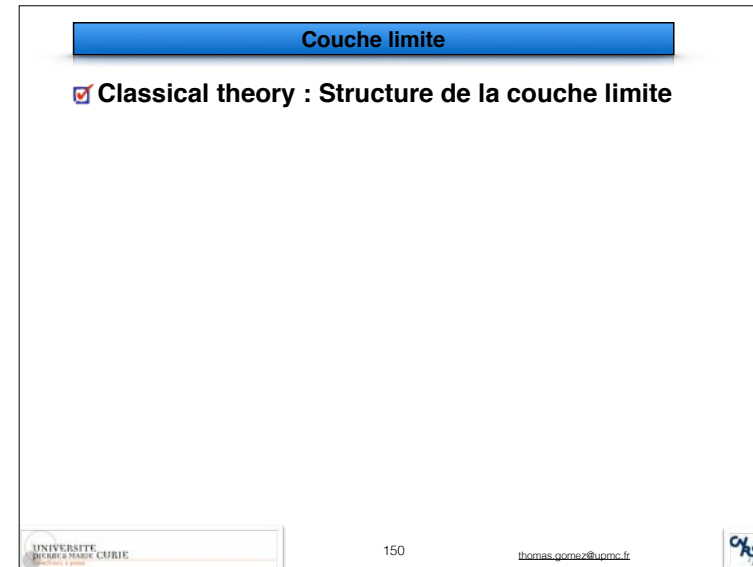
Production d'anisotropie

$$y^+ \geq 12$$

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### Couche limite

Classical theory : Structure de la couche limite

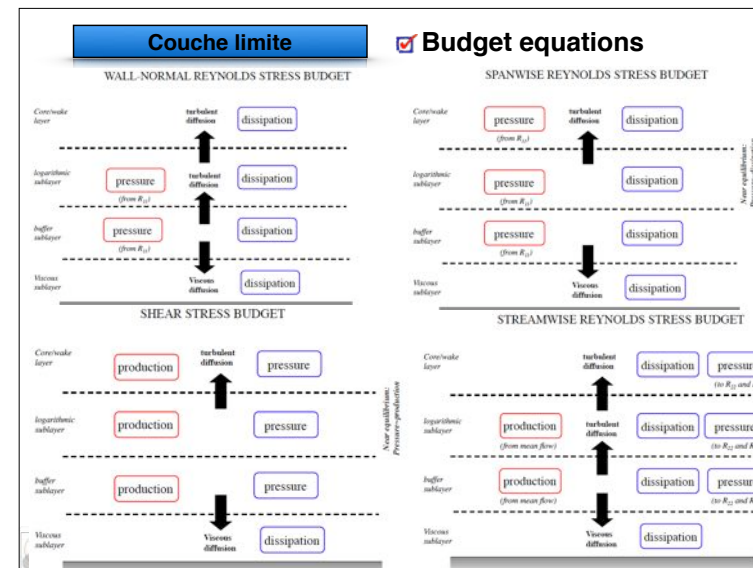
Layer	Regions	$\bar{u}(y)$
viscous sublayer	$0 \leq y^+ \leq 3 - 5$	$\bar{u}^+ = y^+$
buffer layer	$3 - 5 \leq y^+ \leq 30 - 50$	empirical law
logarithmic layer	$30 - 50 \leq y^+ \leq 0, 1\delta^+$	$\bar{u}^+ = \frac{1}{\kappa} \ln y^+ + C_1$
logarithmic sublayer	$30 - 50\nu/u_* \leq y \leq 0, 1\delta$	$\bar{u} = \frac{1}{\kappa} \ln(y/h) + C_2$
wake	$y \geq 0, 1\delta$	empirical law

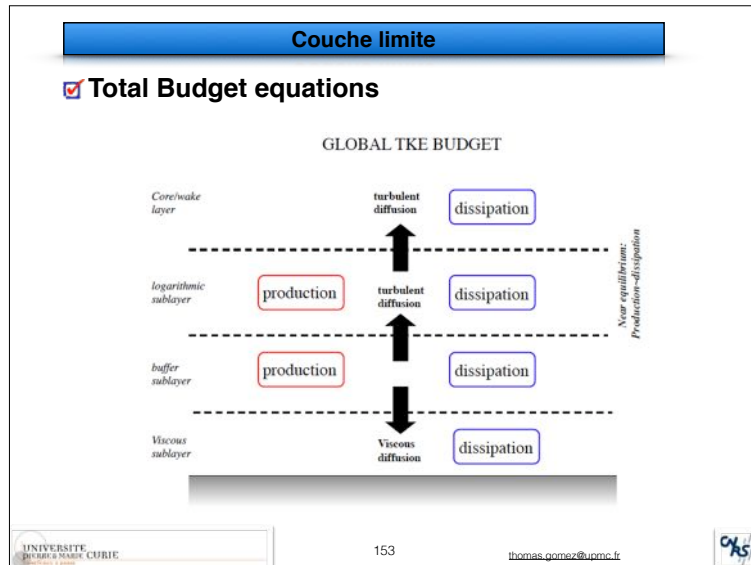
Region	Simplified turbulent kinetic energy budget
viscous sublayer	dissipation = viscous diffusion
buffer layer	production = turbulent diffusion + dissipation
logarithmic layer	production = dissipation
wake	turbulent diffusion = dissipation

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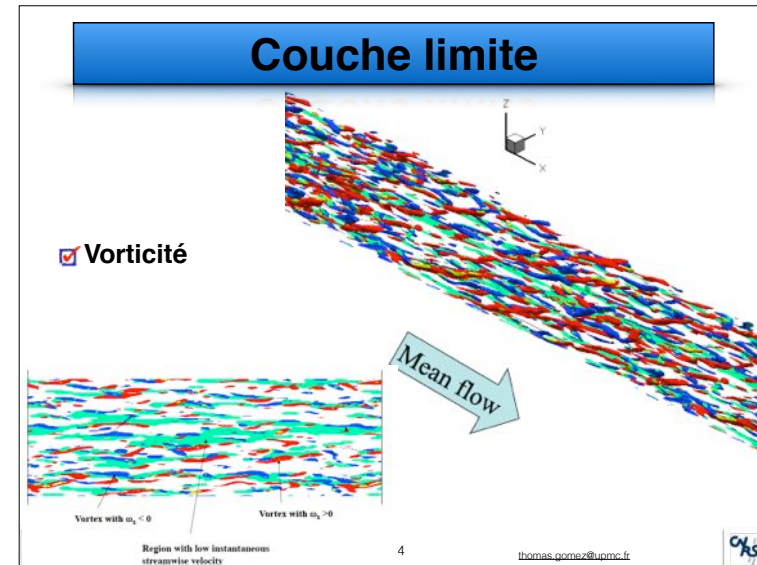
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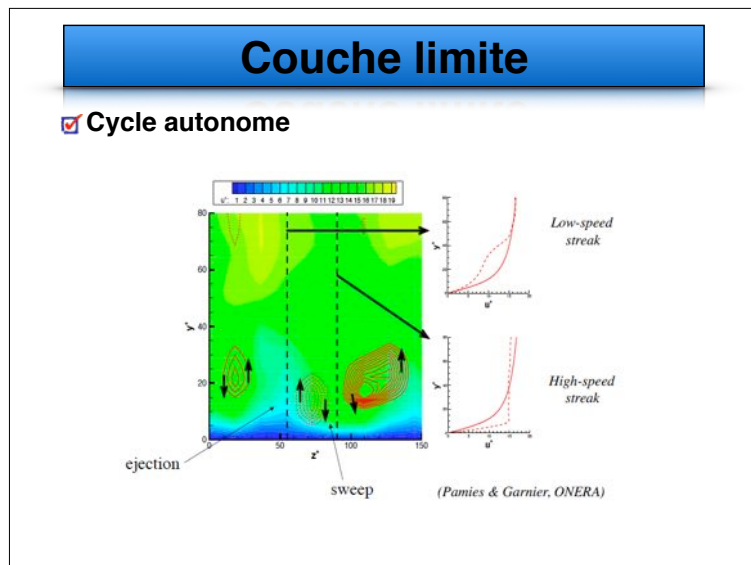
163



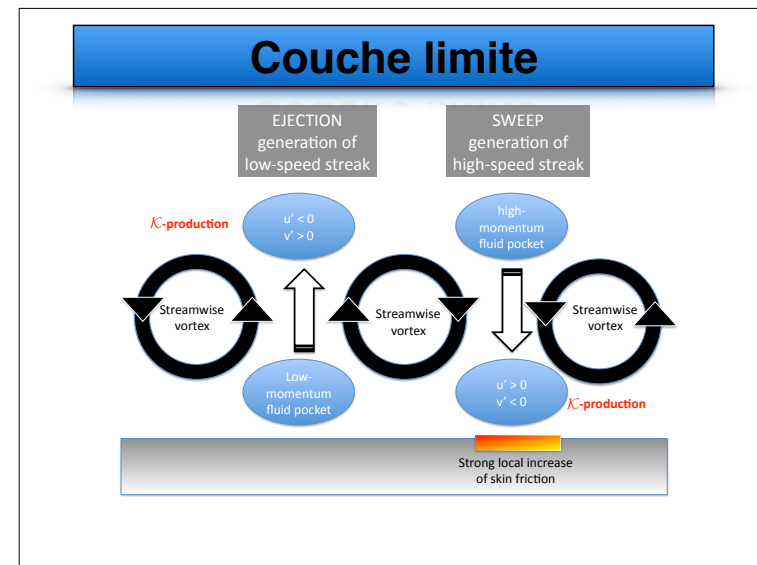
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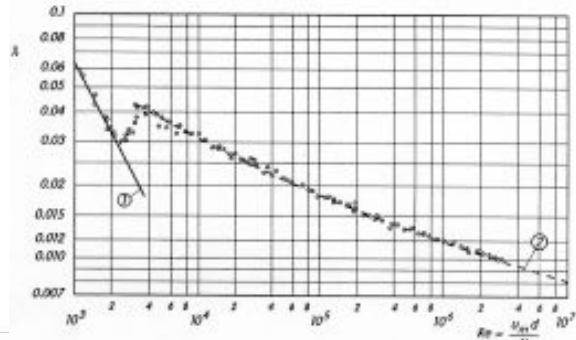


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## Couche limite

### ☑ Coefficient de trainée

$$C_f = u_*^2 / \frac{1}{2} \bar{u}_b^2$$



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## Couche limite

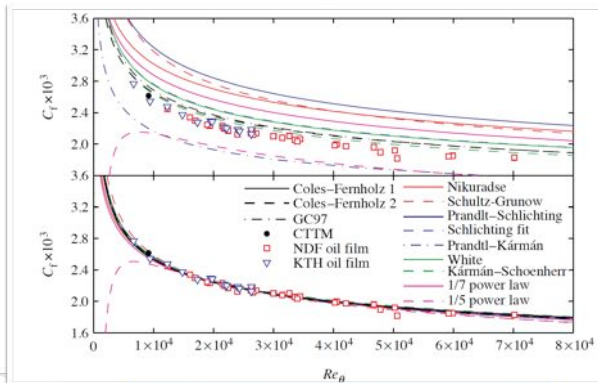
### ☑ Lois semi-empiriques

Relation	Forme originale	Modifications
Coles-Fernholz 1	$C_f = 2[1/\kappa_{VR} \ln(Re_{\delta^*}) + C^*]^{-2}$	$\kappa_{VR} = 0,384, C^* = 3,354$
Coles-Fernholz 2	$C_f = 2[1/\kappa_{VR} \ln(Re_{\theta}) + C]^{-2}$	$\kappa_{VR} = 0,384, C = 4,127$
Karman-Schoenherr	$C_f = 0,558 C'_f / [0,558 + 2(C'_f)^{-1/2}]$	
	$C'_f = [\log(2Re_{\theta}) / 0,242]^{-2}$	0,2385
Prandtl-Schlichting	$C_f = 0,455 (\log Re_x)^{-2,58} - A/Re_x$	0,3596
Prandtl-Karman	$C_f^{-1/2} = 4 \log(Re_x \sqrt{C_f}) - 0,4$	2,12
Schultz-Grunow	$C_f = 0,427 (\log Re_x - 0,407)^{-2,64}$	0,3475
Nikuradse	$C_f = 0,02666 Re_x^{-0,139}$	-0,1502
Schlichting	$C_f = (2 \log Re_x - 0,65)^{-2,3}$	-2,3333
White	$C_f = 0,455 [\ln(0,06 Re_x)]^{-2}$	0,4177
Loi 1/7	$C_f = 0,027 Re_x^{-1/7}$	0,02358
Loi 1/5	$C_f = 0,058 Re_x^{-1/5} - A/Re_x$	0,0655
George-Castillo	$C_f^{1/2} = 2(55/C_{i\infty} [\delta^+]^{-7\infty} \exp[A/(\ln \delta^+)^{\alpha}])$	56,7

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## Couche limite

### ☑ Lois semi-empiriques ajustées

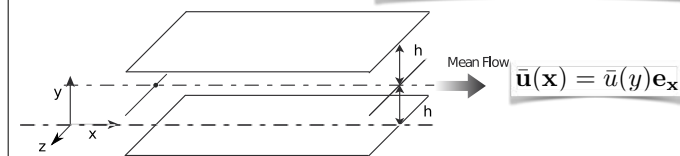


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## Couche limite

### ☑ Trainée : Lois non locales exactes (FIK)

Fukagata Iwamoto Kasagi



#### ☑ Hyp :

- ☑ Débit constant
- ☑ Homogène dans la direction transverse z
- ☑ Moments statistiques symétriques par rapport au plan de symétrie y=h

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## Couche limite

**Trainée : Lois non locales exactes**

$$-\frac{\partial}{\partial x} \bar{p} = \frac{\partial}{\partial y} \left[ R_{12} - \frac{1}{Re_b} \frac{\partial}{\partial y} \bar{u} \right] + \frac{\partial}{\partial t} \bar{u} + \underbrace{\frac{\partial}{\partial x} \bar{u}\bar{u} + \frac{\partial}{\partial y} (\bar{u}\bar{v})}_{I_x} - \frac{1}{Re_b} \frac{\partial^2}{\partial y^2} \bar{u}$$

$$\phi''(x, y, t) \equiv \bar{\phi}(x, y, t) - \tilde{\phi}(x, t), \quad \tilde{\phi}(x, t) \equiv \int_0^1 \bar{\phi}(x, y, t) dy$$

Inhomogénéité suivant la direction x

$$C_f = 12 \left[ \frac{1}{Re_b} - \int_0^1 2(1-y)R_{12}(y)dy + \frac{1}{2} \int_0^1 (1-y^2) \left( I_x'' + \frac{\partial p''}{\partial x} + \frac{\partial}{\partial t} \bar{u} \right) dy \right]$$

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## Couche limite

**Lois non locales exactes**

$$C_f = 12 \left[ \frac{1}{Re_b} - \int_0^1 2(1-y)R_{12}(y)dy + \frac{1}{2} \int_0^1 (1-y^2) \left( I_x'' + \frac{\partial p''}{\partial x} + \frac{\partial}{\partial t} \bar{u} \right) dy \right]$$

**Hyp : Stationnaire et homogène suivant x**

Configuration	Relation
Plane channel	$C_f = 12 \left[ \frac{1}{Re_b} - \int_0^1 2(1-y)\overline{u'v'} dy \right]$
Pipe flow	$C_f = 16 \left[ \frac{1}{Re_b} - \int_0^1 2r\overline{u'_r u'_z} dr \right]$
ZPG Boundary layer	$C_f = 4 \left[ \frac{(1-\delta_d)}{Re_\delta} - \int_0^1 (1-y)\overline{u'v'} dy \right]$

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## Couche limite

**Utilisation des lois FIK : Cas compressible**

Contributions Compressible/Turbulente

$$C_f = \frac{6}{Re} + \underbrace{6 \int_{-1}^0 z(\rho)(u'w') dz}_{c_r} + \underbrace{6 \int_{-1}^0 -z(\bar{\mu}) \frac{\partial(u')}{\partial z} dz}_{c_c} + \underbrace{6 \int_{-1}^0 -z \left( \mu' \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right) \right) dz}_{c_{cr}}$$

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## Couche limite

**Utilisation des lois FIK => Etude de stratégies de réduction de trainée**

- Aspiration/Succion
- Surfaces hydrophobes

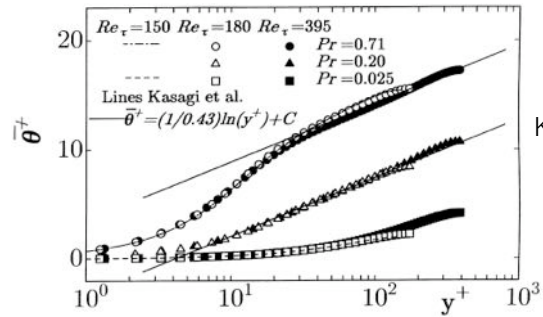
Nano-Tubes

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## Couche limite thermique

☑ Scalaire passif

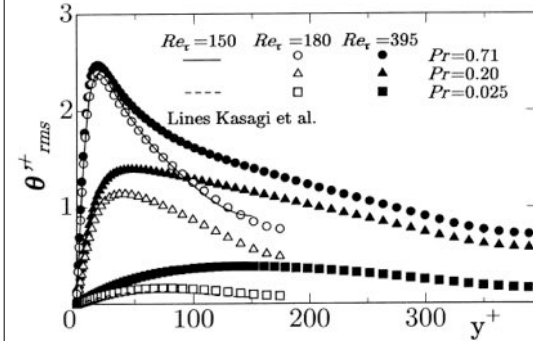


Kawamura et al.  
1999

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## Couche limite thermique

☑ Scalaire passif

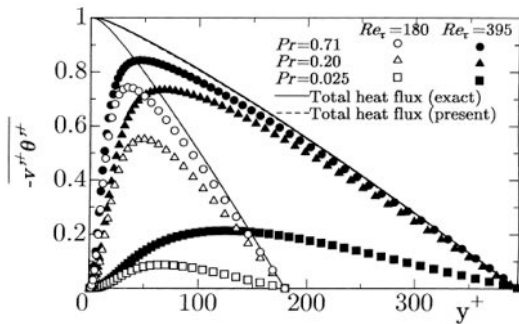


Kawamura et al.  
1999

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## Couche limite thermique

☑ Flus de scalaire passif



Kawamura et al.  
1999

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## Couche limite thermique

☑ Transfert de chaleur à la paroi

Nombre de Stanton

$$c_h = \frac{q_w}{c_p \rho u_c (\bar{T}_w - \bar{T}_c)}$$

Nombre de Nusselt

$$Nu \equiv c_h Pr Re_c$$

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# Couche limite thermique

☑ Scalaire passif

Nombre de Stanton

$$c_h = \frac{q_w}{c_p \rho u_c (\bar{T}_w - \bar{T}_c)}$$

Nombre de Nusselt

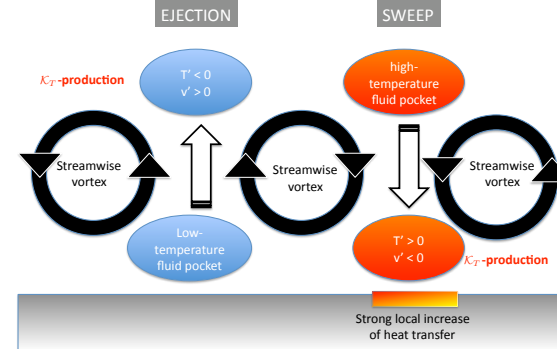
$$Nu \equiv c_h Pr Re_c$$

Configuration	Relation
Plane channel (CTD)	$Nu = \left[ 2 - Re_b Pr \int_0^1 v' T' dy \right]$
Plane channel (CHF)	$\frac{1}{Nu} = \left[ \frac{17}{70} - \frac{1}{2} \int_0^1 (1-\phi)(-v'T') dy - \frac{1}{2} \int_0^1 ((y^3 - 3y^2 + 1)\phi_d - \phi_d^2) dy \right]$ $+ \frac{1}{Re Pr} \int_0^1 u' T' dy$
Plane channel (UHG)	$\frac{1}{Nu} = \left[ \frac{1}{5} - \frac{1}{2Q} \int_0^1 (1-\phi)(-v'T') dy - \int_0^1 (1-y)\phi_d dy \right]$ $+ \frac{1}{Re Pr} \int_0^1 u' T' dy$

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# Couche limite thermique

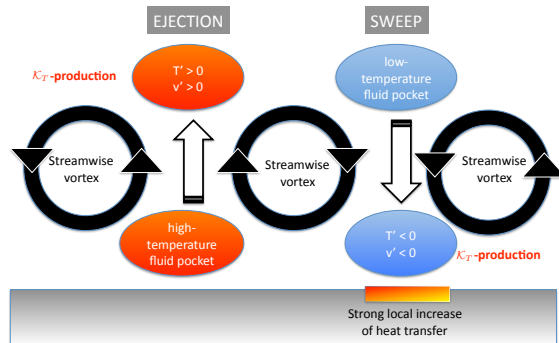
Cold wall case  $\frac{\partial \bar{T}}{\partial y} > 0$



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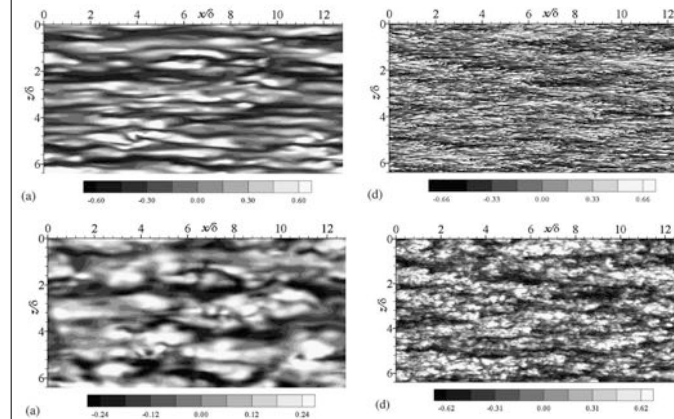
# Couche limite thermique

Hot wall case  $\frac{\partial \bar{T}}{\partial y} < 0$



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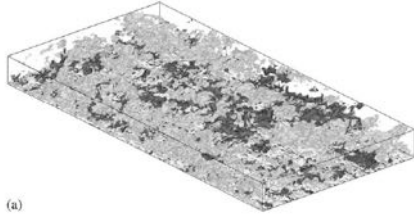
# Couche limite thermique



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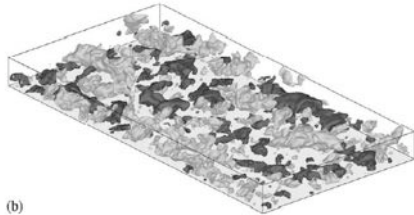
## Couche limite thermique

☑ Champs instantanés de fluctuations de température



$$Pr = 0.71$$

(a)



$$Pr = 0.025$$

(b)

Abe et al. 2004