HOW TO USE THIS UMAT

1. Outline of the Madou-Leblond (ML) model

- The ML model is an extension of the classical Gurson model for plastic porous materials incorporating void shape effects. The voids are schematized as ellipsoidal with three distinct axes. The model thus also extends that of Gologanu, Leblond and Devaux which considered spheroidal voids only (with two distinct axes).

- The ML yield criterion reads

$$\frac{Q(\boldsymbol{\sigma})}{\bar{\sigma}^2} + 2q(1+g)(f+g)\cosh\left(\frac{L(\boldsymbol{\sigma})}{\bar{\sigma}}\right) - (1+g)^2 - q^2(f+g)^2 = 0$$

  where

  * $Q(\boldsymbol{\sigma})$ is a certain quadratic form of the stress tensor $\boldsymbol{\sigma}$;
  * $L(\boldsymbol{\sigma})$ is a certain linear form of the diagonal components of $\boldsymbol{\sigma}$;
  * f is the porosity;
  * g is the "second porosity";
  * q is Tvergaard's parameter;
  * $\bar{\sigma}$ is the "average yield stress" of the sound matrix.

- The plastic flow rule is associated to the yield criterion via the normality property.

- The model also includes evolution equations of the internal parameters: it predicts the evolution in time of the porosity, the shape and orientation of the voids, and the state of hardening.

- Hardening is described in the way suggested by Gurson: it is assumed to be of isotropic type and represented by the sole variable $\bar{\sigma}$; this variable is identical to $\sigma(\bar{\varepsilon})$ where $\sigma(\varepsilon)$ is the current yield stress of the sound material, depending on the equivalent cumulated strain ε, and $\bar{\varepsilon}$ is the "average cumulated strain" of the plastic matrix; and $\bar{\varepsilon}$ obeys the same evolution equation as in Gurson's model.

- Tvergaard's parameter q is modified in a specific way according to the shape of the voids.

- Coalescence of voids is included in a simple, heuristic way, by replacing $f+g$ in the yield criterion and the flow rule by the quantity $(f+g)^*$ defined by

$$(f+g)^* = \begin{cases} f+g & \text{if } f+g \le (f+g)_c \\ (f+g)_c + \delta[f+g-(f+g)_c] & \text{if } f+g > (f+g)_c \end{cases}$$

  where $(f+g)_c$ and δ are parameters. This modelling exactly parallels that suggested by Tvergaard and Needleman for Gurson's model.

- Unilateral contact between the voids' surface and the embedded inclusions, or auto-contact between the voids' faces, may be accounted for by introducing minimum values $a^{\min}, b^{\min}, c^{\min}$ of the voids' major, intermediate and minor semi-axes a, b, c. The modelling is approximate in that, although the voids' semi-axes are prevented from taking values smaller than the minimum ones prescribed, the effect of contact upon the yield criterion and the associated plastic flow rule is disregarded.

- Continuous nucleation of voids can also be included in the manner suggested by Chu and Needleman, by including in the porosity rate $\dot{f}$ an extra term $\dot{f}_n$ given by

$$\dot{f}_n = \frac{f_n^{\max}}{s_n\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\bar{\varepsilon} - \varepsilon_n}{s_n}\right)^2\right]\dot{\bar{\varepsilon}}$$

  where $f_n^{\max}$, ε_n and s_n are material parameters. Any coupling between nucleation and evolution of the shape and orientation of the voids is disregarded, that is the newly created voids are assumed to appear with semi-axes identical in size and orientation to those of the pre-existing voids.

- Nahshon and Hutchinson's modification of the evolution equation of the porosity is *not* included in the model. Indeed the aim of this heuristic modification is to account, within Gurson's model, for the damage arising from the change of shape of the voids; but this effect is already included in the ML model and does not require any extra ingredient.

- Papers defining the model are included in the package provided. The two papers published in the **Journal of the Mechanics and Physics of Solids** define the yield criterion (and the associated flow rule). The first paper published in the **European Journal of Mechanics A/Solids** provides a validation of this criterion through micromechanical finite element simulations, and the second a definition of the evolution equations of the shape and orientation of the voids.

2. How to run the programme

2.1. Generalities

- The calculations must be run within a 3D option. There is no 2D version of the ML model.

- Use C3D8 elements or similar ones.

- The UMAT uses some routines of the LAPACK library. Such routines can be freely downloaded at the web address http://www.netlib.org/lapack.

- The file STRINGS.inc (containing strings of characters for error messages) must be included in the same directory as the UMAT.

2.2. Input of model data

- The name of the material must be MADLEB; otherwise an error message is issued and the calculation is stopped.

- The number of state variables must be 9; otherwise an error message is issued and the calculation is stopped.

- The number of material parameters must be at least 22: 2 elastic constants, 16 parameters pertaining to voids (see below), and at least 4 plastic parameters (2 points on the uniaxial stress-strain curve of the sound material). Otherwise an error message is issued and the calculation is stopped.

- Material parameters must be introduced in the following order, without any omission:

  $E,\ \nu,\ f_0,\ P^0(\mathbf{u})$ (6 components), $q,\ (f+g)_c,\ \delta,\ a^{\min},\ b^{\min},\ c^{\min},\ f_n^{\max},\ \varepsilon_n,\ s_n,\ \varepsilon_1,\ \sigma_1,\ \varepsilon_2,\ \sigma_2,\ \ldots$

  where E is Young's modulus, ν Poisson's ratio, f_0 the initial porosity, $P^0(\mathbf{u})$ the quadratic form characterizing the initial shape and orientation of the voids (see below), $q, (f+g)_c, \delta, a^{\min}, b^{\min}, c^{\min}, f_n^{\max}, \varepsilon_n, s_n$ have been defined above, and $(\varepsilon_1, \sigma_1)$, $(\varepsilon_2, \sigma_2)$, etc. are the successive points on the uniaxial stress-strain curve of the sound material.

- The initial porosity f_0 must not be zero. (A zero value will result in an overflow in subroutine CRITML). If the porosity is generated by some continuous nucleation process with a zero initial value, input a very small value for f_0.

- The quadratic form $P(\mathbf{u})$ characterizing, at all instants, the shape and orientation of the voids is defined by the formula

$$P(\mathbf{u}) = \frac{(\mathbf{u}.\mathbf{e}_x)^2}{a^2} + \frac{(\mathbf{u}.\mathbf{e}_y)^2}{b^2} + \frac{(\mathbf{u}.\mathbf{e}_z)^2}{c^2}$$

  where $\mathbf{u}$ denotes an arbitrary vector, $a \geq b \geq c$ the semi-axes of the voids, and $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ the unit vectors parallel to the directions of these semi-axes. Note that O denoting the centre of a void and M the current point on its surface, this surface is defined by the equation $P(\mathbf{OM}) = 1$; and that the eigenvalues of the matrix $\mathbf{P}$ of the quadratic form $P(\mathbf{u})$ are a^{-2}, b^{-2}, c^{-2}, whereas its normalized eigenvectors are $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$. The components of the matrix $\mathbf{P}^0$ of the initial quadratic form $P^0(\mathbf{u})$ in the fixed analyst's frame (O, x_1, x_2, x_3) (not the voids' frame!) must be entered in the order $P^0_{11}, P^0_{22}, P^0_{33}, P^0_{12}, P^0_{13}, P^0_{23}$. For instance, to define initially spherical voids of radius a,

input the numbers $\frac{1}{a^2}, \frac{1}{a^2}, \frac{1}{a^2}, 0, 0, 0$. To define ellipsoidal voids of initial semi-axes a_1, a_2, a_3 parallel to the directions x_1, x_2, x_3 of the analyst's frame, input the numbers $\frac{1}{a_1^2}, \frac{1}{a_2^2}, \frac{1}{a_3^2}, 0, 0, 0$. (Note that a_1, a_2, a_3 here need not coincide with a, b, c in this order, that is, the inequalities $a_1 \geq a_2 \geq a_3$ need not be satisfied).

- Although the programme requires initial values of the voids' semi-axes and calculates their evolution in time, the results are insensitive to multiplication of these semi-axes by a given constant (because the ML model does not include any intrinsic material lengthscale). Thus one of the initial semi-axes may conventionally be set to unity.

- Do not input zero values for the diagonal components of the matrix $\mathbf{P}^0$, because the programme does not accommodate infinite semi-axes, that is cylindrical voids. If such voids must be considered, schematize them as ellipsoidal with a very large major semi-axis.

- Since one cannot input infinite values for the diagonal components of $\mathbf{P}^0$, zero semi-axes, that is cracks, are also ruled out. If such voids must be considered, schematize them as ellipsoidal with a very small minor semi-axis.

- To disregard coalescence of voids, input any value for $(f + g)_c$ and a value of unity for δ. Do *not* input zero values for $(f + g)_c$ and δ.

- To account for contact between the voids' surface and the embedded inclusions, input positive, finite values for $a^{\min}, b^{\min}, c^{\min}$. To account for auto-contact between the voids' faces, input very small but nonzero values for $a^{\min}, b^{\min}, c^{\min}$.

- To disregard contact between the voids' surface and the embedded inclusions, and auto-contact between the voids' faces, input zero values for $a^{\min}, b^{\min}, c^{\min}$. Note that the semi-axes may then become negative. In such an instance a message error is issued and the calculation is stopped.

- To disregard continuous nucleation, input zero values for $f_n^{\max}$, ε_n, s_n.

- In the pairs (ε, σ) defining the uniaxial stress-strain curve of the sound material, ε is the logarithmic plastic strain (total logarithmic strain minus elastic strain), and σ the Cauchy stress (force divided by present area).

- The first plastic strain ε_1 must be zero, whereas the first stress σ_1 (the initial yield stress) must be nonzero; otherwise an error message is issued and the calculation is stopped.

- The number of pairs (ε, σ) is limited to 100; if this value is exceeded an error message is issued and the calculation is stopped.

2.3. Method of solution

- At the present stage of development of the programme the tangent matrix is not calculated; the output DDSDDE of the UMAT is simply the elastic stiffness matrix. Therefore, *do not use a Newton method to solve the nonlinear equations of the problem*, because the programme would just use the elastic stiffness matrix instead of the true tangent matrix. *Use a quasi-Newton (BFGS) method*, which has proved robust in all the cases tested, although the number of iterations required may not be small.

3. How to interpret the results

- In addition to standard quantities (displacements, stresses, etc.), 9 internal variables (SDV1 to SDV9) are calculated and stored by ABAQUS, and available for examination and postprocessing. These internal variables are, in this order:

$$f,\ \ P(\mathbf{u})\ \text{(6 components)},\ \ \bar{\varepsilon},\ \ d.$$

- The components of the matrix **P** of the quadratic form $P(\mathbf{u})$ in the analyst's frame (O, x_1, x_2, x_3) are provided in the order $P_{11}, P_{22}, P_{33}, P_{12}, P_{13}, P_{23}$. Except if the matrix is diagonal, these components do *not* directly provide the present semi-axes a, b, c and orientation of the voids. To obtain these, a small postprocessing (not provided in the UMAT) is necessary, in the form of a diagonalization of the 3×3 symmetric matrix **P**. The semi-axes of the voids are the inverse square roots of the eigenvalues of **P**, and their orientation is defined by the eigenvectors of this matrix.

- The hardening parameter $\bar{\varepsilon}$ is conventionally stored with a minus sign in the case of elastic unloading.

- The parameter

$$d = \frac{q(f+g)^*}{1+g}$$

  is not a true internal variable of the model, and is provided only for the sake of examination and postprocessing. This parameter characterizes the state of damage of the material; when it is zero the material is sound (the yield criterion reduces to that of von Mises), and when it is unity the material is completely destroyed (the elastic domain reduces to the point $\boldsymbol{\sigma} = \mathbf{0}$, as can be seen by inputting this value of $\boldsymbol{\sigma}$ into the yield criterion). Plotting isocontours of d is useful to visualize damaged zones, for instance cracks.

4. How to modify the UMAT

- Care has been taken to include numerous comments into the programme, so as to facilitate modifications.

- In order to allow for easy incorporation of the programme into *any* finite element code, a “core programme” PROJML with specific arguments has been written; the UMAT consists only of a modest “interface” transforming ABAQUS’s arguments into the arguments of PROJML, calling PROJML, and transforming back the modified arguments of PROJML into those of ABAQUS. Thus, adapting the programme to another finite element code only requires to modify the interface, not the core programme. If, on the other hand, modifications of the model are required, the essential modifications must be made in PROJML. A few simple examples follow.

- Comment lines 1613 to 1616 of PROJML to suppress the modification of Tvergaard’s parameter q depending on the void shape.

- Comment lines 2135 to 2138 of PROJML to suppress any evolution of the shape and orientation of the voids. With initially spherical voids, this is equivalent to performing the calculation with Gurson’s model disregarding void shape effects. This option may be useful to quickly compare the predictions of the ML model to those of Gurson’s model, without using another UMAT.

- Modify lines 2178 to 2180 of PROJML to calculate the consistent tangent matrix. If you do so, activate also the various lines of the programme marked “CC”.