SUMMARY OF SCIENTIFIC WORKS

Jean-Baptiste Leblond

March 11, 2014

Contents

1

General presentation

My scientific activities are mainly concerned with three areas of the mechanics of deformable solids:

- The numerical simulation of welding processes, together with the theoretical study of the associated problems of material behaviour.
- The mechanics of brittle fracture, in both two and three dimensions.
- The ductile fracture of metals.

My interests also encompass a number of ancillary themes. Some of these, like the anisotropic damage of concrete and the fatigue of elastomers, again pertain to the mechanics of deformable solids. Other themes belong to related areas. The study of the simultaneous diffusion and precipitation of chemical elements in metallic matrices, for instance, represented an occasion to extend my interests to some problems of metallurgy and mathematics.

My taste is inclined toward the beauty and harmony of rigorous theoretical approaches. In the field of brittle fracture mechanics, the foundations of which are well established, my satisfaction lies in the elegance, and sometimes the difficulty of analytical methods of solution. In other fields where modelling plays a more central role, I like to adopt rigorous homogenization procedures when the microscopic mechanisms responsible for the phenomenon to be modelled are well understood and sufficiently simple to allow for such an approach. However, when the microscopic mechanisms are not well understood or too complex to be amenable to some homogenization procedure, I do not hesitate to adopt a purely phenomenological approach.

In addition to personal taste, my connections with the industrial world have always played a major role in the definition of my scientific interests. I have had cooperations with ESI Group (since 1980), Framatome-Areva, EdF, Arcelor-Mittal and Michelin.

2

Abstracts of some typical papers

Remark: The abstracts are grouped thematically. The papers are denoted by the symbols used in the full list of publications. A few typical illustrations are also provided.

Transformation plasticity

Mathematical modelling of transformation plasticity in steels - I: Case of ideal-plastic phases [Pub:LDD89] (with J. DEVAUX and J.C. DEVAUX)

Transformation plasticity in steels (i.e., the anomalous plastic flow observed during the progress of a phase transformation) is usually attributed to two distinct physical mechanisms, which have been proposed by Greenwood and Johnson and Magee. This paper proposes a theoretical approach to the problem, in the case where the Magee mechanism is negligible and the phases are ideal-plastic. An explicit expression for the transformation plastic strain rate is obtained for a steel undergoing a transformation under a small applied stress; this expression is consistent with experiments conducted on various materials. A finite element simulation provides a confirmation of the theoretical formula and allows for a detailed examination of the validity of some physical hypotheses made in the treatment. It also allows for a study of transformation plasticity under high applied stresses. Based on these results, a general (i.e., applicable for all kinds of stresses applied) model is proposed in the case of ideal-plastic phases.

Mechanics of ductile rupture

Analytical study of a hollow sphere made of porous plastic material and subjected to hydrostatic tension - Application to some problems in ductile fracture of metals [Pub:PL90] (with G. PERRIN)

A solution to the problem stated in the title is given by treating the porous matrix as a homogeneous material obeying the Gurson-Tvergaard model. As a first application, it is shown that the flow stress under hydrostatic loading of a material containing two populations of voids with different sizes is almost the same as that of a material with only one population of voids and the same total porosity. As a second application, a self-consistent method is used to derive a value for the parameter introduced by Tvergaard in the original Gurson model to account for interactions between cavities. Other models are also considered and shown to fail to satisfy the self-consistency requirement, whatever the value chosen for the parameter characterizing interactions between voids.

Rudnicki and Rice's analysis of strain localization revisited [Pub:PL93] (with G. PERRIN)

In a celebrated paper published in 1975, Rudnicki and Rice (RR) analyzed the conditions for strain localization in pressure-sensitive dilatant materials. This paper corrects certain results in their work that are relevant in exceptional cases. RR found that the normal to the plane of localization was generally orthogonal to the direction of the intermediate principal stress; however, when a certain inequality was satisfied, it was orthogonal to the direction of the minimum principal stress. In contrast, it is shown here that the normal to the plane of localization is always orthogonal to the direction of the intermediate principal stress; but there is a special case where it is parallel to the direction of either the maximum or the minimum principal stress, and in this case the expression of the critical hardening modulus at localization differs from that given by RR.

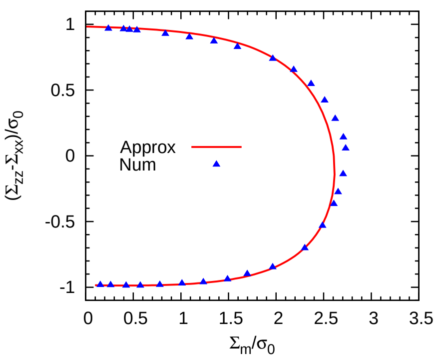
Recent extensions of Gurson's model for porous ductile metals [Book:GLPD97] (with M. GOLOGANU, G. PERRIN and J. DEVAUX)

This paper is devoted to two distinct extensions of Gurson's (1977) famous model for plastic voided metals. Gurson's work was based on an approximate limit-analysis of a typical elementary volume in a porous material, namely a hollow sphere subjected to conditions of arbitrary homogeneous boundary strain rate. The first extension envisaged consists of considering a more general geometry, namely a spheroidal volume containing some spheroidal confocal cavity. The aim here is to incorporate void shape effects into Gurson's model. The second extension again considers a hollow sphere, but now subjected to conditions of inhomogeneous boundary strain rate. The goal is to account for possible strong variations of the macroscopic mechanical fields at the scale of the representative cell (i.e. of the void spacing), as encountered near crack tips.

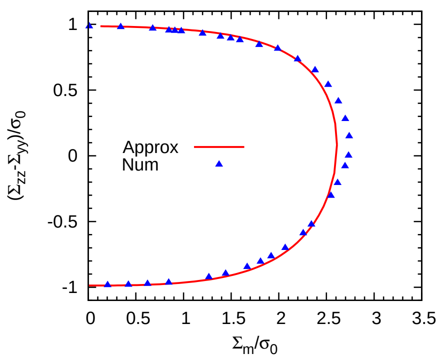
A Gurson-type criterion for porous ductile solids containing arbitrary ellipsoidal voids - I: Limit-analysis of some representative cell; II: Determination of yield criterion parameters [Pub:ML12a] and [Pub:ML12b] (with K. MADOU)

Part I: Gurson's (1997) famous model of the behavior of porous ductile solids, initially developed for spherical cavities, was extended by Gologanu *et al.* (1993, 1994, 1997) to spheroidal, both prolate and oblate voids. The aim of this work is to further extend it to general (non-spheroidal) ellipsoidal cavities, through approximate homogenization of some representative elementary porous cell. As a first step, we perform in the present Part I a limit-analysis of such a cell, namely an ellipsoidal volume made of some rigid-ideal-plastic von Mises material and containing a confocal ellipsoidal void, loaded arbitrarily under conditions of homogeneous boundary strain rate. This analysis provides an estimate of the overall plastic dissipation based on a family of trial incompressible velocity fields recently discovered by Leblond and Gologanu (2008), satisfying conditions of homogeneous strain rate on all ellipsoids confocal with the void and the outer boundary. The asymptotic behavior of the integrand in the expression of the global plastic dissipation is studied both far from and close to the void. The results obtained suggest approximations leading to explicit approximate expressions of the overall dissipation and yield function. These expressions contain parameters the full determination of which will be the object of Part II.

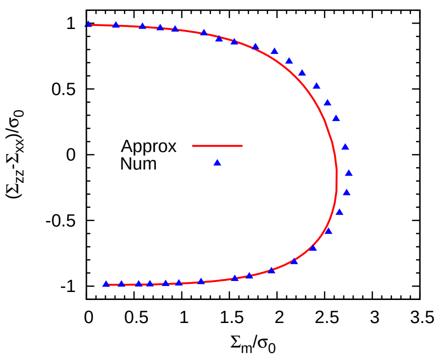
Part II: The aim of this paper is to fully determine the parameters of the approximate homogenized yield criterion for porous ductile solids containing arbitrary ellipsoidal cavities proposed in Part I. This is done through improvements of the limit-analysis of some representative hollow cell presented there. The improvements are of two kinds. For hydrostatic loadings, the limit-analysis is refined by performing micromechanical finite element computations in a number of significant cases, so as to replace Leblond and Gologanu's (2008) trial velocity field representing the expansion of the void by the exact, numerically determined one. For deviatoric loadings, limit-analysis is dropped and direct use is made of some general rigorous results for nonlinear composites derived by Ponte-Castaneda (1991), Willis (1991) and Michel and Suquet (1992) using the earlier work of Willis (1977) and the concept of "linear comparison material". This hybrid approach is thought to lead to the best possible expressions of the yield criterion parameters. The criterion proposed reduces to (variants of) classical approximate criteria proposed by Gurson (1977) and Gologanu *et al.* (1993, 1994, 1997) in the specific cases of spherical or spheroidal, prolate or oblate cavities. An overview of the validation of this criterion through micromechanical finite element computations is finally presented.



(a) $\Sigma_{xx} = \Sigma_{yy} \neq \Sigma_{zz}$, $\Sigma_{xy} = \Sigma_{yz} = \Sigma_{zx} = 0$



(b) $\Sigma_{xx} = \Sigma_{zz} \neq \Sigma_{yy}$, $\Sigma_{xy} = \Sigma_{yz} = \Sigma_{zx} = 0$



(c) $\Sigma_{yy} = \Sigma_{zz} \neq \Sigma_{xx}$, $\Sigma_{xy} = \Sigma_{yz} = \Sigma_{zx} = 0$

Figure 2.1: Traces in three orthogonal planes of the yield locus for a void with semi-axes in the proportions 10:2:1 and a porosity of 0.01 - Numerical results (Num) and approximation proposed (Approx).

Effective yield criterion accounting for microvoid coalescence [Pub:BL13] (with A. BENZERGA)

An effective yield function is derived for a porous ductile solid near a state of failure by microvoid coalescence. Homogenization theory combined with limit analysis are used to that end. A cylindrical cell is taken to contain a coaxial cylindrical void of finite height. Plastic flow in the intervoid matrix is described by J_2 theory while regions above and below the void remain rigid. Velocity boundary conditions are employed which are compatible with an overall uniaxial straining for the cell, a postlocalization kinematics that is ubiquitous during the coalescence of neighboring microvoids in rate-independent solids. Such boundary conditions are not of the uniform strain rate kind, as is the case for Gursonlike models. A similar limit analysis problem for a square-prismatic cell containing a square-prismatic void was posed long ago (Thomason, P. F., 1985, "Three-dimensional models for the plastic limit-loads at incipient failure of the intervoid matrix in ductile porous solids", Acta Metallurgica, vol. 33, pp. 1079-1085). However, to date a closed-form solution to this problem has been lacking. Instead, an empirical expression of the yield function proposed therein has been widely used in the literature. The fully analytical expression derived here is intended to be used concurrently with a Gursonlike yield function in numerical simulations of ductile fracture.

A new numerical implementation of a second-gradient model for plastic porous solids, with an application to the simulation of ductile rupture tests [Pub:BLP14] (with J.M. BERGHEAU and G. PERRIN)

An interesting second-gradient model for plastic porous solids, extending Gurson's (1977) standard first-gradient model, was proposed by Gologanu *et al.* (1997) in order to settle the issue of unlimited localization of strain and damage and the ensuing mesh sensitivity in finite element calculations. Gologanu *et al.*'s (1997) model was implemented in a finite element code by Enakoutsa and Leblond (2007, 2009). The implementation however rose two difficulties: (i) the number of degrees of freedom per node was awkwardly large because of the introduction of extra nodal degrees of freedom representing strains; (ii) convergence of the global elastoplastic iterations was often very difficult.

A new implementation solving these problems is presented in this work. An original procedure of elimination of the nodal degrees of freedom representing the strains permits to reduce the number of degrees of freedom per node to its standard value. Also, the convergence issue is solved through use of normally integrated linear elements instead of more customary subintegrated quadratic ones.

As an application, 2D numerical simulations of experiments of ductile rupture of a pre-notched and pre-cracked axisymmetric specimen and a CT specimen are performed. The calculations are pursued without difficulties up to a late stage of the rupture process, and the results are mesh-independent. Also, a good agreement between experimental and computed load-displacement curves is obtained for values of the parameters governing void coalescence compatible with those suggested by micromechanical numerical simulations, which could never be achieved in calculations based on Gurson's (1977) standard model.

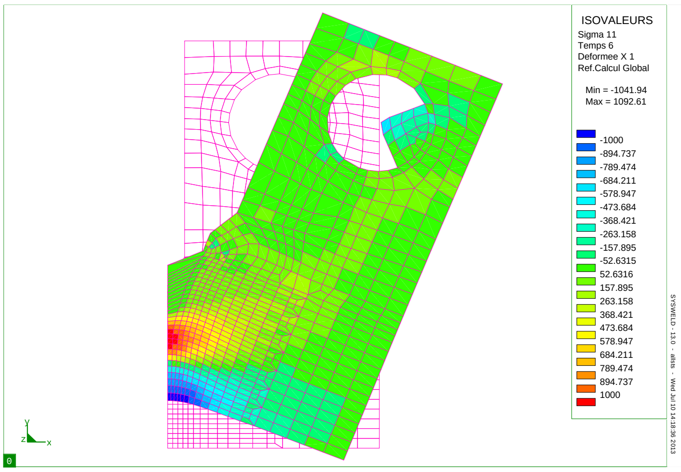


Figure 2.2: Distribution of the axial stress on the deformed configuration of the CT specimen at time $t = 6$ s.

2D brittle fracture mechanics

Crack kinking from an initially closed, ordinary or interface crack, in the presence of friction [Pub:LF04] (with J. FRELAT)

One theoretically studies crack kinking from an ordinary crack (in some homogeneous material) or an interface crack (between two dissimilar materials), in the situation where this crack is closed prior to kinking but open after it. This problem was recently considered by the authors with the simplifying, but physically quite unrealistic hypothesis of absence of friction between the crack lips. Their work is extended here to account for possible friction governed by Coulomb's law. Problems of elastic fracture mechanics with unilateral contact and friction between the crack lips being not only non-linear, but incremental in nature, the theoretical treatment becomes notably more involved than without friction. It is still based, however, on the same basic ingredients, namely "homogeneity" properties of the type of problems considered, changes of scale and some reasonable hypotheses. It is shown that whatever the geometry of the body and the crack and whatever the loading, the asymptotic expression of the stress intensity factors (SIF) at the tip of a vanishingly small kinked crack extension depends solely upon the initial (mode II) SIF prior to kinking, the kink angle, Dundurs's famous parameters α and β and the friction coefficient. The (history-independent) functions involved in the general formulae established are determined numerically though finite element computations. From there, using Goldstein and Salganik's famous *principle of local symmetry* to predict the crack path, one derives a theoretical value for the kink angle. This value depends upon the loading only through the sign of the initial stress intensity factor; it also depends on the mismatch of elastic properties and the friction coefficient. However, its range of variation is numerically found to be rather narrow. Experiments conducted by various authors seem to confirm these theoretical predictions.

3D brittle fracture mechanics

The tensile tunnel-crack with a slightly wavy front [Pub:LMP96] (with S.E. MOUCHRIF anf G. PERRIN)

Several problems of three-dimensional fracture mechanics for a planar tunnel-crack loaded in pure mode I in an infinite elastic solid are investigated. The first is that of the bifurcation from the fundamental straight configuration of the crack front in brittle fracture, i.e. the possibility of the appearance of another, curved configuration still possessing the property that the stress intensity factor be constant along the front. The second is the stability of the same fundamental configuration versus small deviations from straightness within the crack plane in fatigue. The third is the (analytic at the start, but numerical in fine) determination of the fundamental "kernel" appearing in the integral expression of the variation of the stress intensity factor induced by a small perturbation of the crack front; this topic is considered after (and not before as would seem more natural) the first two in order to illustrate the fact that investigating the latter does not require a precise knowledge of that kernel as a necessary prerequisite. The last question envisaged is (again analytical first, but finally numerical) the calculation of the crack-face weight function in mode I for the crack configuration envisaged. Gao and Rice's previous works (1985, ASME J. Appl. Mech. vol. 52, pp. 571-519; 1986, ASME J. Appl. Mech. vol. 53, pp. 774-778; 1987, Int. J. Fracture vol. 33, pp. 155-174; 1987, ASME J. Appl. Mech. vol. 54, pp. 627-634; 1988, Int. J. Solids Structures vol. 24, pp. 177-193) devoted to other crack shapes have been an important source of inspiration for this study with regard to both the topics investigated and some of the methods used.

Theoretical analysis of crack front instability in mode I+III [Pub:LKL11] (with A. KARMA and V. LAZARUS)

This paper focusses on the theoretical prediction of the widely observed crack front instability in mode I+III, that causes both the crack surface and crack front to deviate from planar and straight shapes, respectively. This problem is addressed within the classical framework of fracture mechanics where the crack front evolution is governed by conditions of constant energy-release-rate (Griffith criterion) and vanishing stress intensity factor of mode II (principle of local symmetry) along the front. The formulation of the linear stability problem for the evolution of small perturbations of the crack front exploits previous results of Movchan *et al.* (1998) (suitably extended) and Gao and Rice (1986), which are used to derive expressions for the variations of the stress intensity factors along the front resulting from both in-plane and out-of-plane perturbations. We find exact eigenmode solutions to this problem, which correspond to perturbations of the crack front that are shaped as elliptic helices with their axis coinciding with the unperturbed straight front and an amplitude exponentially growing or decaying along the propagation direction. Exponential growth corresponding to unstable propagation occurs when the ratio of the unperturbed mode III to mode I stress intensity factors exceeds some "threshold" depending on Poisson's ratio. Moreover, the growth rate of helical perturbations is inversely proportional to their wavelength along the front. This growth rate therefore diverges when this wavelength goes to zero, which emphasizes the need for some "regularization" of crack propagation laws at very short scales. This divergence also reveals an interesting similarity between crack front instability in mode I+III and well-known growth front instabilities of interfaces governed by a Laplacian or diffusion field.

On the strong influence of imperfections upon the quick deviation of a mode I+III crack from coplanarity [Pub:LL14] (with V. LAZARUS)

This work explores the possibility that quick deviations of cracks loaded in mode I+III from coplanarity may be greatly facilitated by inevitable fluctuations of the fracture toughness. The idea is that such fluctuations must induce in-plane undulations of the crack front resulting, because of the presence of the mode III load, in non-zero values of the local stress intensity factor of mode II, implying future local out-of-plane deviations of the crack which might be "unstable" in Cotterell and Rice's (1980) sense if the local non-singular stress parallel to the direction of propagation is positive.

Exploration of this idea implies evaluation of the variations of the local stress intensity factors and non-singular stresses arising from a slight but otherwise arbitrary in-plane perturbation of a semi-infinite crack. These quantities were calculated in works of Rice (1985), Gao and Rice (1986) and Gao (1992) but the evaluation of the non-singular stresses was incomplete, and is supplemented here by using the theory of 3D weight functions (Rice, 1985, Bueckner, 1987).

Inspection of the results shows that for in-plane sinusoidal undulations of the crack front of sufficient (though still small) amplitude, the conditions of nonzero local stress intensity factor of mode II and positive local non-singular stress parallel to the direction of propagation are simultaneously met on some parts of the front, implying the possibility of future local deviations of the crack from coplanarity "unstable" in Cotterell and Rice's (1980) sense, and thus confirming the idea investigated.

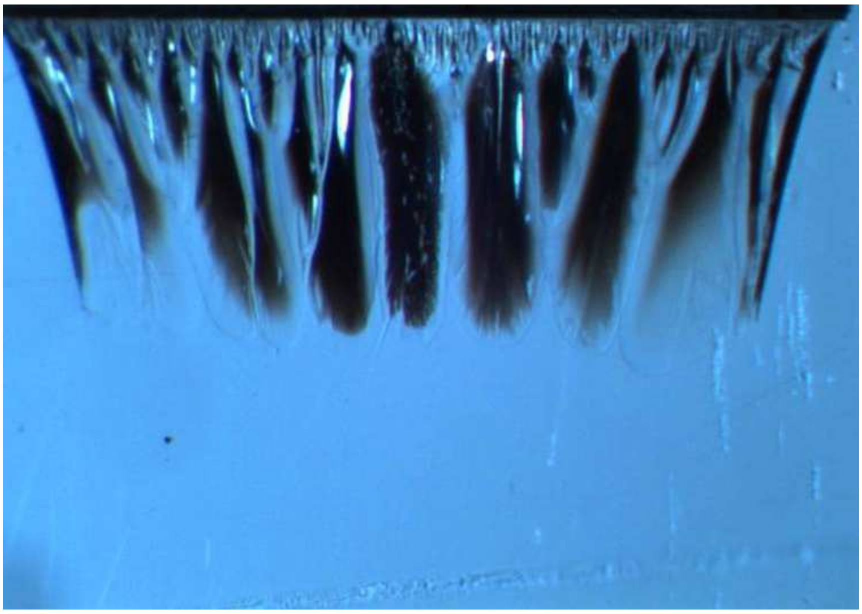


Figure 2.3: Fracture facets in mode I+III - Experiment by Buchholz, photograph by Lazarus.

Coplanar perturbation of a crack lying on the mid-plane of a plate [Pub:LPLFLV11] (with L. LEGRAND, S. PATINET, J. FRELAT, V. LAZARUS and D. VANDEMBROUCQ)

Several groups have studied experimentally the deformation of the front of mode I cracks propagating quasistatically along the interface between bonded plates. The theoretical interpretation of such experiments has always been based up to now on a formula of Rice (1985); this formula provides the first-order variation of the local mode I stress intensity factor resulting from some small, but otherwise arbitrary coplanar perturbation of the front of a semi-infinite crack in an infinite body. To be applicable to bonded plates, this formula requires that the characteristic distance of variation of this perturbation in the direction of the crack front be small compared to all other characteristic dimensions of the problem, and first of all the thickness of the plates. This condition is unfortunately frequently violated in practice. The purpose of this paper is therefore to provide a more exact formula for the variation of the local stress intensity factor, for the specific cracked geometry and boundary conditions used in experiments; this should allow for more accurate theoretical interpretations. This is done in two steps. The first one consists in adapting Rice's (1985) treatment, applicable to the extreme case of plates of infinite thickness, to the other extreme one of plates of infinitesimal thickness, using the standard Love-Kirchhoff plate theory. An interesting outcome of the analysis is that the distance from the crack front to the boundary of the plate acts as a "cutoff length", in the sense that when the distance between two points on the crack front gets larger than it, the influence of the crack advance at the first point upon the stress intensity factor at the second diminishes quickly; the plate thickness, however, plays no similar role. The second step consists in supplementing the theoretical expressions applicable to extreme values of the plate thickness with finite element computations providing results for intermediate values. These computations lead to the definition of a simple, approximate but accurate "interpolation formula" for the variation of the local stress intensity factor, applicable to plates of arbitrary thickness.

A geometrically nonlinear analysis of coplanar crack propagation in some heterogeneous medium [Pub:VLP13] (with M. VASOYA and L. PONSON)

In a recent paper (Legrand *et al.*, 2012), we established, using some results of Rice (1989), the second-order expression of the variation of the mode I stress intensity factor resulting from some small, but otherwise arbitrary coplanar perturbation of the front of a semi-infinite tensile crack in an infinite body. The aim of the present work is to apply the expression found to a geometrically nonlinear analysis of quasistatic, coplanar crack propagation in some heterogeneous medium. In a first step, we recall Legrand *et al.*'s (2012) formula, extending it to the case where the unperturbed stress intensity factor, for the straight configuration of the front, depends on the position of this front; in addition to being intrinsically interesting, such an extension is necessary in order to avoid meaningless divergent integrals in what follows. In a second step, assuming the local energy-release-rate to be equal everywhere on the crack front to its critical value, we derive an expression of the shape of this front accurate to second order in the fluctuations of toughness of the material. In a third step, as an application, we present a second-order calculation of the equilibrium shape of the crack front, when it penetrates a single infinitely elongated obstacle or a periodic distribution of such obstacles. Special attention is paid to the case, of particular physical interest, where the derivative of the unperturbed stress intensity factor with respect to the position of the crack front can be neglected.

Internal oxidation

Numerical simulation of internal oxidation of steels during annealing treatments [Pub: FHL05] (with P. FLAUDER and D. HUIN)

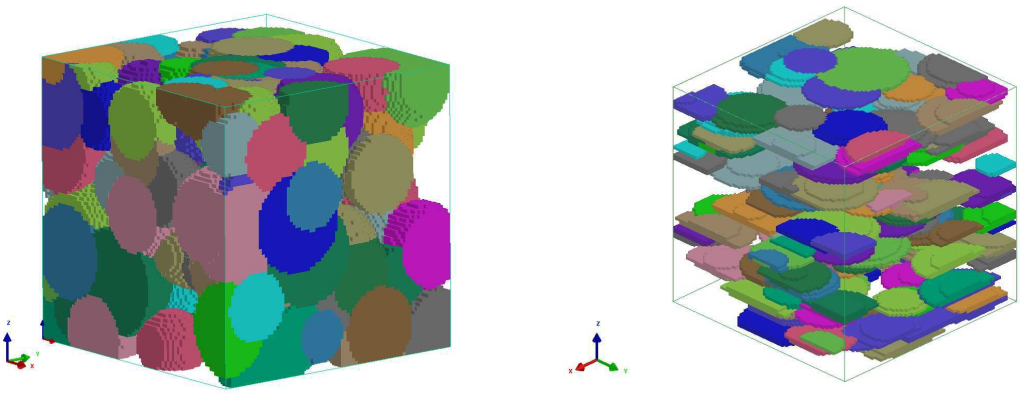
This paper presents a new model for simultaneous diffusion and precipitation of chemical elements in metallic matrices, a scheme for its numerical solution, and several applications to problems of internal oxidation. The model stands as an extension of the classical Wagner model for internal oxidation of steels, but is much more general in that it allows for an arbitrary number of diffusing chemical elements, an arbitrary number of precipitate phases with arbitrary compositions, dependence of diffusion coefficients and solubility products upon (time-dependent) temperature, etc., thus allowing for a much broader range of applications. As a counterpart, it is generally impossible to solve the complex non-linear equations of the model analytically, but this can be done numerically. The simple but efficient numerical scheme proposed is based on explicit 1D finite differences. Experience has shown that this scheme, in spite of its rusticity and the restrictions it imposes on the time-step, is more efficient than more elaborate strategies based on the finite element method. The applications presented are concerned with internal oxidation of steels during annealing treatments. The model and associated numerical scheme allow for evaluation of the amounts of the various oxide precipitates in the external layer of the sheet. This opens the way, through numerical parametric studies of the influence of the process parameters and the chemical composition, to the improvement of existing treatments and the development of new steel grades.

Mathematical results for a model of diffusion and precipitation of chemical elements in solid matrices [Pub:L05]

This paper discusses mathematical results for a classical model of simultaneous diffusion and precipitation of chemical elements in metallic matrices. This model consists of diffusion equations coupled with the laws of mass action, which express the hypothesis of instantaneous thermodynamic equilibrium between the matrix and precipitate phases. Existence and uniqueness of the solution of the local problem of thermodynamic equilibrium are established, but the full boundary value problem is shown to be prone to severe mathematical pathologies which are interpreted as limitations of the model due to some oversimplifying physical assumptions. An improved version of the model is proposed and shown to fit within the framework of Amann's theory for systems of strongly coupled, quasilinear equations of parabolic type; Amann's results ensure existence, uniqueness and smoothness of maximal solutions. It is also shown that positivity of solutions is ensured provided that it is guaranteed initially and on the boundary of the domain. However, the problem of global existence of a solution remains open.

Predicting the transition from internal to external oxidation of alloys using an extended Wagner model [Pub:LPH13] (with M. PIGNOL and D. HUIN)

Leblond (2011) recently estimated the conditions governing the transition from internal to external oxidation of alloys using a variant of Wagner's (1959) model incorporating the possible role of oxides as diffusion barriers, through a heuristic dependence of the diffusion coefficient of oxygen upon their local volume fraction. But the crudeness of the formula adopted made the prediction of the onset of external oxidation only qualitative. A more accurate formula is derived here by using a thermal analogy and finite element computations of the reduction of the conductivity generated by nonconducting, more or less flat obstacles. The extended Wagner model incorporating this formula leads to a prediction of the "critical" local fraction of oxides corresponding to the transition from internal to external oxidation, depending on the "aspect ratio" of the oxides. The predicted value is in acceptable agreement with that measured by Rapp (1961) for the Ag-In system, for a reasonable postulated value of this aspect ratio.



(a) Aspect ratio 1 (spheres)   (b) Aspect ratio 10 (very flat spheroids)

Figure 2.4: Representative cells containing spheroidal obstacles (diffusion in the vertical direction).

Damage of concrete

Application of some anisotropic damage model to the prediction of the failure of some complex industrial concrete structure [Pub:BGL07] (with P. BADEL and V. GODARD)

The aim of this paper is to present a new model for the anisotropic damage of concrete and apply it to the numerical simulation of the failure of some complex structure. This model considers damage as anisotropic in tension but isotropic in compression, and incorporates asymmetry between tension and compression. In spite of its complexity, it is expressed in a format fit for numerical calculations by the finite element method, and involves only six material parameters (in addition to the usual elastic constants), which makes it suitable for applications to large industrial structures. The material parameters can be identified from simple uniaxial and biaxial experiments. The application envisaged relates to the failure of some cylindrical nuclear containment vessel subjected to some excessive internal pressure. This structure is made of reinforced concrete containing both passive (initially stress-free) steel armatures and pre-stressed cables. The numerical results illustrate the importance of incorporation of both asymmetry between tension and compression and anisotropy of damage in the simulation to accurately predict the resistance of the structure to fracture.

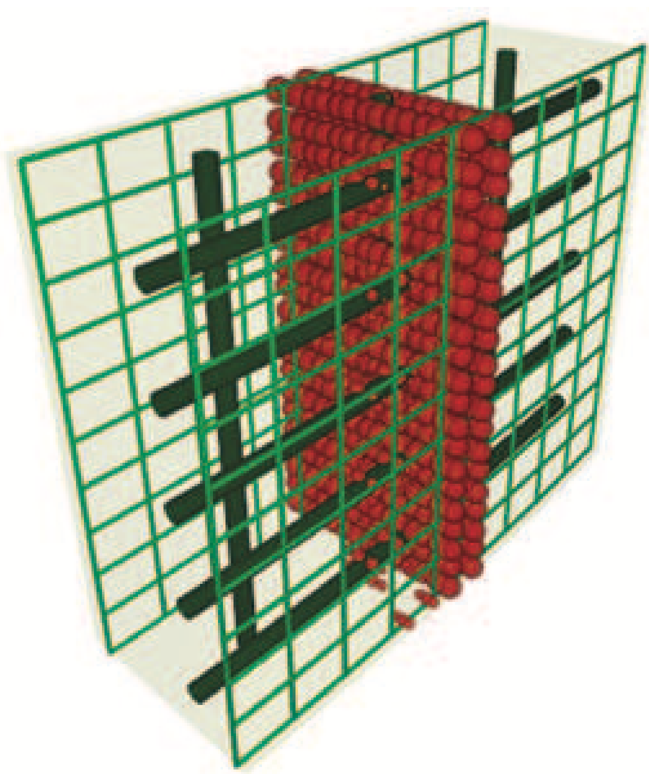


Figure 2.5: Damage of a confining vessel made of reinforced concrete and subjected to some excessive internal pressure. The thin light green beams represent the passive (initially stress-free) steel armatures, and the thick dark green ones the pre-stressed cables. The red spheres symbolize damage; their radius is proportional to the damage variable.

Fatigue of elastomers

On the heuristic extension of Haigh's diagram for the fatigue of elastomers to arbitrary loadings [Pub:BGL09] (with J.B. BRUNAC and O. GERARDIN)

Haigh's diagram provides the fatigue lifetime (number of cycles to failure) of an elementary volume of material subjected to some cyclic loading, as a function of the maximum and minimum stresses (or strains) reached during each cycle. It is determined experimentally for uniaxial loadings only, so that using it in more complex, three-dimensional situations requires to extend it heuristically by defining, for each arbitrary periodic loading, some "equivalent" uniaxial loading. The approach proposed in this paper is inspired in principle, if not in details, from Dang Van's works on the definition of a criterion for non-initiation of fatigue cracks in metals subjected to arbitrary cyclic loadings. Its principle basically consists of determining the smallest sphere containing an arbitrary closed stress path in the stress space, and approximately characterizing this path through two parameters only related to the center and the radius of this sphere. These parameters are then heuristically used as entries of Haigh's diagram to predict the fatigue lifetime. Multiaxial fatigue experiments performed by Mars and Saintier are used to critically assess this proposal, and it is found to improve upon previous ones.

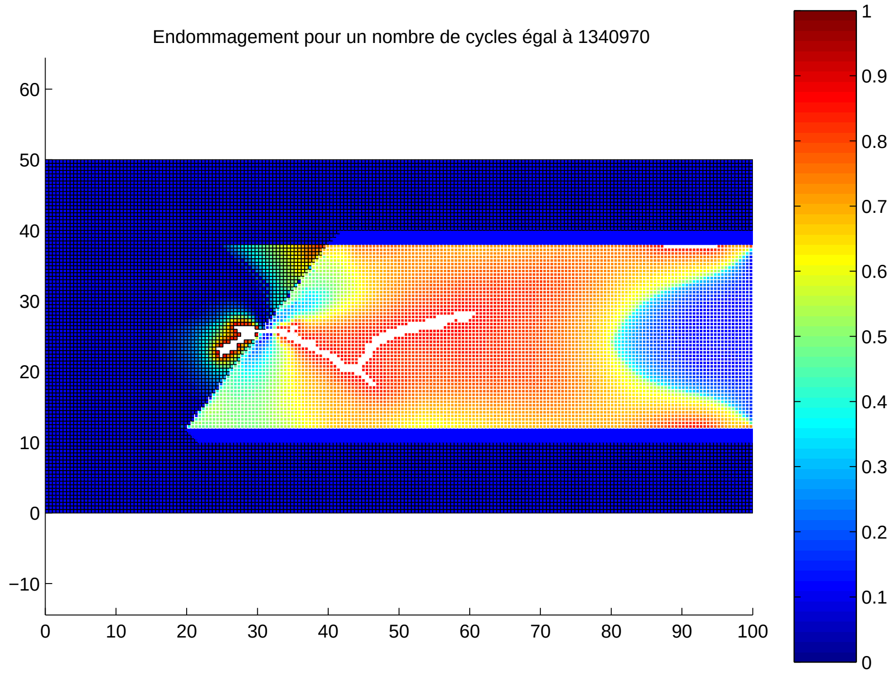


Figure 2.6: Propagation of a fatigue crack in some heterogeneous elastomer (after 1,340,970 cycles). The horizontal light blue bands are quasi-rigid and the zone between these bands is less stiff than the rest of the structure.